AXISVM

AUTO N$_{cr}$ FLEXURAL BUCKLING COEFFICIENT CALCULATION

Intentionally blank page

TABLE OF CONTENTS

1. INTRODUCTION

The rapid increase of available computational resources is driving fundamental changes in the applied methodologies for structural design. Structural engineers expect not only more efficient calculation but also smarter software that can perform a growing number of design tasks automatically. Although automated design is faster, it is not necessarily more economical than the conventional approach. Economy heavily depends on the applied design methodology and the supplied inputs. If a task is out of the scope of the applied methodology, or inappropriate inputs are supplied, the error in results might become unacceptable. Note that such an error is not necessarily on the conservative side, thus it might have dire consequences.

A popular area of automation in structural engineering design environments is the standardized verification of structural member resistance. In Europe this typically corresponds to strength and stability checks according to the Eurocode standards. A subset of the parameters required for these verifications can be derived from the structural model (e.g. cross-section sizes, element length, material properties, etc.). The remaining parameters need to be provided by the user (e.g. load introduction point, concrete reinforcement layout, connection size and details, etc.) Determination of the user-defined parameters such as the flexural buckling coefficient, often involve engineering judgement and information that is not available in the structural model. Therefore, it is difficult to replace such user input with a reliable automated calculation.

Although several structural engineering design environments offer automatic calculation of the flexural buckling coefficient, their implementation is often limited to basic cases. Users unaware of these limits are prone to making significant mistakes in their designs. The automatic buckling coefficient calculation tool in AxisVM (in the following referred to as the AutoN_{cr} Tool for brevity) has been developed with an emphasis on large scope and transparent operation. The next section explains the scope and objectives of the AutoN_{cr} Tool so that AxisVM users can decide if and when it can be of assistance. The following section on the methodology explains the working mechanism of the algorithm to provide a better understanding of its applicability limits. This is followed by numerical examples of increasing complexity. The examples demonstrate cases when the AutoN_{cr} Tool can help as well as others when it has limited applicability.

2. OBJECTIVES AND SCOPE

An automatic buckling coefficient calculation tool might serve two conflicting purposes:

- It saves time.
  A typical example is a simple, but large numerical model with a lot of different members. The user knows the corresponding buckling coefficients for every member, but it takes a lot of time to supply that information to the program. An automatic tool can do this in an instant.

- It deals with complex cases.
  A complicated structural layout or semi-rigid connections are only two examples of the many cases when a structural engineer finds it difficult to determine the buckling coefficient of a member in a reliable manner without advanced analyses. An automatic tool can do the necessary calculations in the background and provide these values conveniently to the user.

The second, complex calculation typically requires additional information from the user before the calculation could begin and the calculation itself needs to use a sophisticated algorithm. Such a tool needs more time for user interaction and for the calculation when compared to a simple tool that can only work with the basic cases. Our primary objective is to find a level of complexity that serves the majority of our users in terms of calculation speed and has a sufficiently broad scope not to hinder its applicability for real-world design. Consequently, the current version of the AutoN_{cr} Tool is not designed to cope with special cases in terms of structural layout, member geometry or load distribution. Future improvements are expected in that area by an extension of the AutoN_{cr} Tool.

Besides the level of complexity, it is also important to specify the kind of answer that we expect from the AutoN_{cr} Tool. In the typical use case we do not expect the user to perform second order analyses to find the buckling resistance of a member, but to apply a simplified methodology based on the elastic critical force (N_{cr}) of the member and reduction factors calibrated to empirical results. Such a methodology is prescribed in the Eurocode 3 (EC3-1-1) standard in EN 1993-1-1 6.3 (CEN, 2009a) for instance. Such methodologies assume that member slenderness is proportional to N_{cr}. This critical force shall be calculated using gross sectional and member properties. Member imperfections, residual stresses and variation of the yield strength are considered by modification factors in the methodology, hence these effects shall not influence the value of N_{cr}.

Linear buckling analysis solves an eigenvalue problem on an ideal linear elastic member. It takes into account the axial load distribution in the member and the restraints presented by boundary conditions or connecting elements. Thus, it yields the N_{cr} required by the aforementioned methodology. The AutoN_{cr} Tool is designed to provide such elastic critical loads (and corresponding flexural buckling coefficients) without the need to perform time consuming linear buckling analyses for the structural model.

It is important to emphasize that modelling errors (e.g. incorrect constraints, connection rigidity, missing structural details) influence results of the AutoN_{cr} Tool as much as they would influence results of a linear buckling analysis. Therefore, the user shall make sure that the numerical model appropriately represents the constraints and connections of the member under consideration. Furthermore, the user is encouraged to use engineering judgement to verify the results of the automatic calculation and perform further analyses if he is not confident with them.

3. METHODOLOGY

The automatic calculation algorithm is based on the recommendation of the European Convention for Constructional Steelwork in their background documentation to the EC3-1-1 standard (ECCS, 2006). The methodology in Annex A of the background documentation has been implemented and enhanced to increase the range of cases handled by the AutoN_{cr} Tool. The following paragraphs introduce the methodology used in AxisVM.

The methodology presented in the background documentation calculates the buckling length of each member of the structure independently on a simplified model. The simplified model is created by extracting the member under investigation from the complex structure and replacing connecting elements with semi-rigid supports. The methodology in the background documentation handles a single column supported at both ends by two perpendicular beams and/or a continuation of the column (*Figure 1*). We extended that approach in two aspects:

- Introduction of supports or connecting elements is not limited to the ends of the investigated member. Any number of connecting elements are allowed at arbitrary points along the length of the member.
- Sufficiently accurate results are expected for cases when elements connect at an arbitrary in-plane angle (β). The centerline of the connecting element does not have to be in the buckling plane. The angle between the element and the buckling plane (α) can also take a large range of values without sacrificing result accuracy (*Figure 2*)

The simplified model solved by the AutoN_{cr} Tool is illustrated in *Figure 1b*. Note that despite the extension of the original approach, the calculation is still based on a simplified model that can only consider the immediate connections of the investigated member. Thus, if distant parts of the structure influence flexural buckling of the investigated member, this simplified approach will not provide an accurate N_{cr} value. The corresponding limitations are explained at the end of this section.
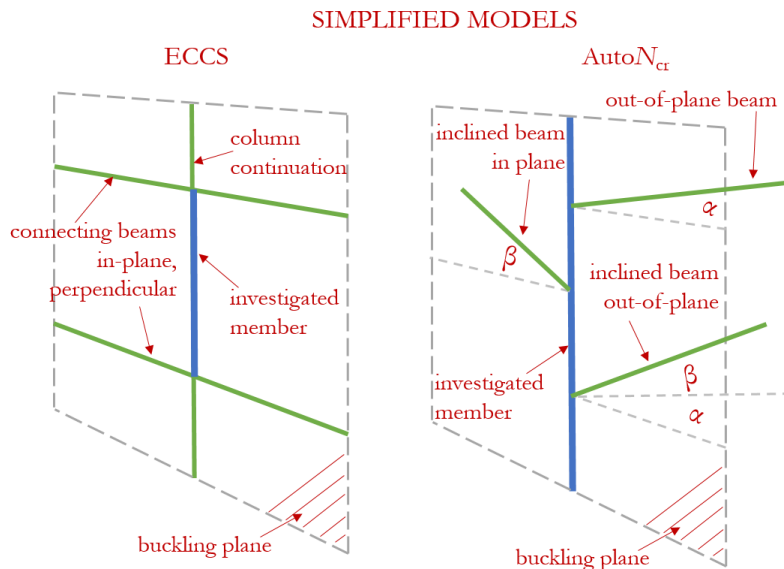

Figure 1 *Comparison of the simplified model and its scope in the ECCS methodology (a) and in AxisVM (b).*
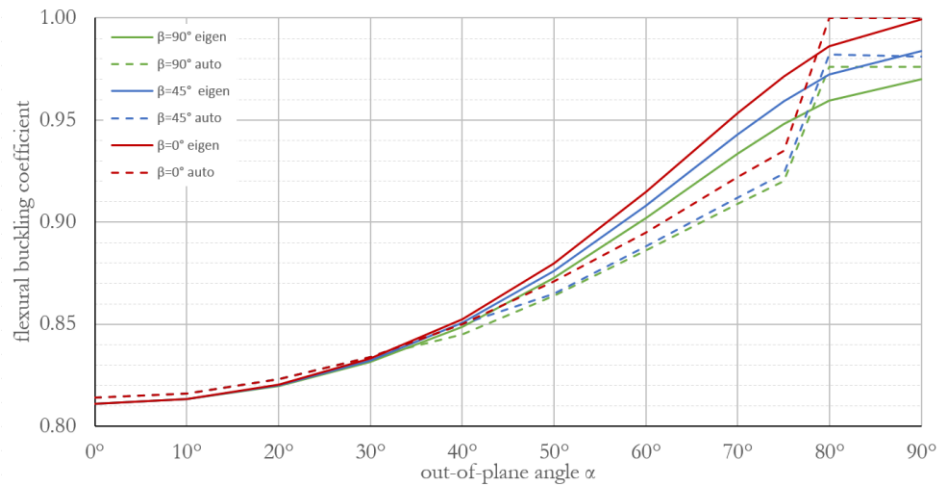
Figure 2 *Accuracy of the AutoN$_{cr}$ Tool in AxisVM for calculation of the flexural buckling coefficient of a column supported by a beam at different angles (see example in 4.3.4 for details). Note that the error in the calculation is negligible for a large range of α and β combinations.*

A support in the simplified model can represent four types of constraints in the global numerical model:

- Constrained Degree of Freedom:
  AxisVM allows its users to constrain any of the six nodal degrees of freedom at any node. Such user-defined constraints are applied in the simplified model as rigid supports.

- Nodal Support:
  Supports with linear behavior installed at any of the member nodes are copied to the simplified model without modification. Nonlinear supports are replaced by linear ones that have stiffness identical to the initial stiffness of the original nonlinear supports. Non-symmetric behavior (supports active only in tension or compression) is not taken into consideration. Such supports are replaced by symmetric ones.

- Element End Release:
  The member in the simplified model has rigid releases, therefore, the original releases are represented as supports at the ends of the member. Fixed, semi-rigid and hinged releases are modeled as rigid, semi-rigid and free supports, respectively. Limited support resistance is not taken into consideration.

- Connecting Element:
  The support an element can provide to another heavily depends on its own support conditions and internal forces. The AutoN$_{cr}$ Tool categorizes connecting elements based on their supporting conditions on their other ends. *Figure 3* summarizes the support configurations considered for connecting elements. The AutoN$_{cr}$ Tool can consider the reduction in supporting stiffness due to axial force in the connecting element. *Figure 4* shows how increasing the axial force in the supporting element reduces the elastic critical load of the supported element. Note the accuracy of the AutoN$_{cr}$ Tool.

BUCKLING CONFIGURATIONS for CONNECTING MEMBERS
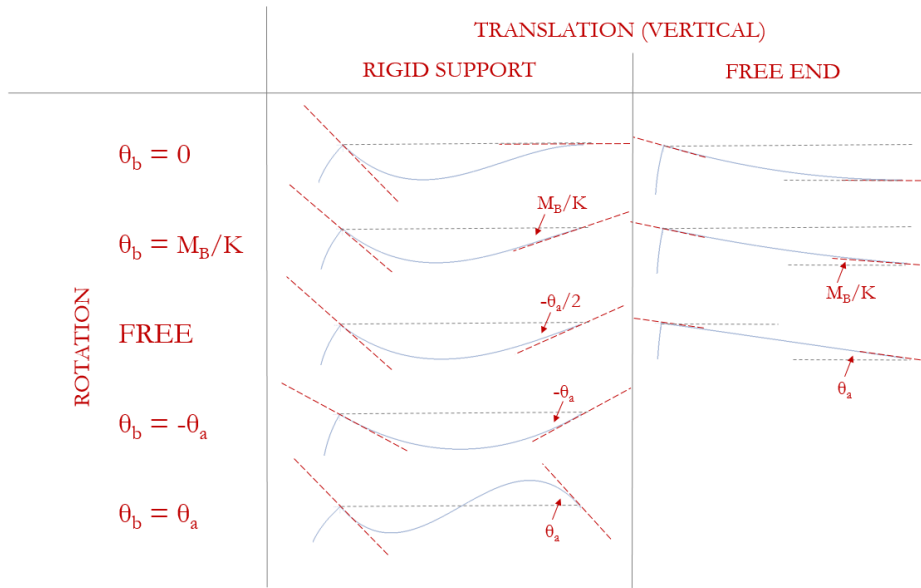
Figure 3 *Support configurations considered for connecting elements in the simplified model.*
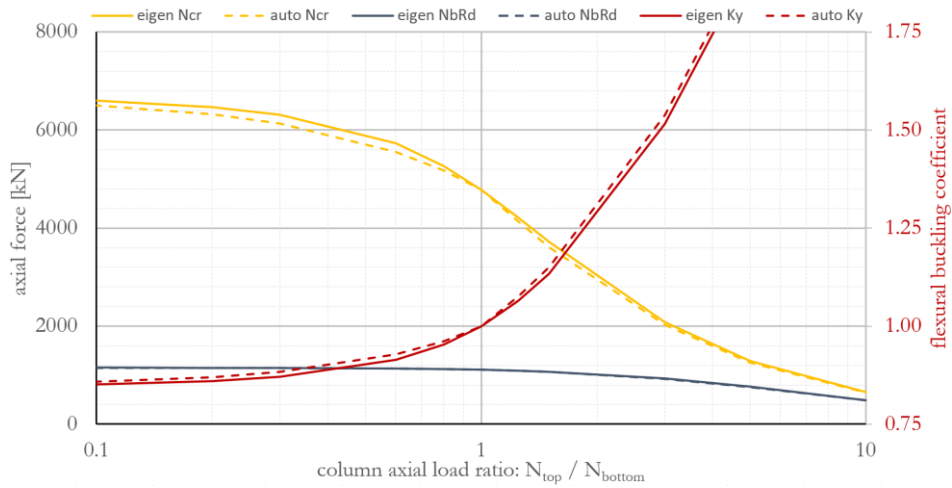
Figure 4 *Elastic critical load of a column as a function of the compression force ratio in its segments (see example in 4.2.5 for details of the corresponding numerical model).*

If a support in the simplified model is made up from several components (e.g. several connecting members and a semi-rigid end release), support stiffness is determined through combination of component contributions.

The flexural buckling coefficient is calculated with expressions from the background documentation (ECCS, 2006). The recommended expression for non-sway buckling mode as per *Eq.(298)*:

$$\frac{L_{cr}}{L} = \frac{1 + 0.145(\eta_1 + \eta_2) - 0.265\eta_1\eta_2}{2 - 0.364(\eta_1 + \eta_2) - 0.247\eta_1\eta_2}$$

The recommended expression for sway buckling mode as per *Eq.(299)*:

$$\frac{L_{cr}}{L} = \sqrt{\frac{1 - 0.2(\eta_1 + \eta_2) - 0.12\eta_1\eta_2}{1 - 0.8(\eta_1 + \eta_2) + 0.60\eta_1\eta_2}}$$

L_{cr} is the buckling length; η_1 and η_2 are the so-called distribution factors that describe the supporting conditions of the member at each end. The distribution factors are calculated from the support stiffnesses in the simplified model with a methodology based on *Eqs.(293)* and *(294)* in the background documentation, but extended to enable the consideration of additional supporting conditions.

Applicability of the presented methodology is limited by the following factors:

- Support stiffnesses in the simplified model are influenced only by the properties of those elements that are directly connected to the member under investigation. Thus, accurate results can only be guaranteed for simple structures that have only a few members. Note that structural members do not buckle individually (with the exception of perfect truss members), but several or all members of the structure lose their stability simultaneously. Thus, flexural buckling of a single member in complex structures can become heavily affected by distant members. The simplified model in the AutoN_{cr} Tool cannot generally take such indirect effects into account.

  Considering that an important area of application is that of frame structures, the AutoN_{cr} Tool is calibrated to provide accurate results for members of frame structures if the following condition is satisfied:

  $$\frac{N_{\mathrm{Ed},m}}{N_{\mathrm{cr},m}} = \frac{N_{\mathrm{Ed},i}}{N_{\mathrm{cr},i}} \quad \text{for all } i$$

  where N_{Ed} is the design axial force; N_{cr} is the elastic critical load assuming a simply supported configuration (i.e. Euler load); m and i indices correspond to the member under investigation and a list of all parallel members in the frame, respectively. This condition is typically satisfied by both columns and beams of conventional frame structures.

- Uniform distribution of axial forces is assumed between consecutive connections of each member. A higher order axial force distribution is replaced by a uniform distribution in the simplified model using the following assumptions: member forces are modeled as the maximum negative value from the non-uniform distribution along the length of the member.
- The stiffening effect of elements under tension is not considered.
- Members are assumed to have uniform cross-sections along their length. Elements with tapered sections are replaced by elements with uniform cross-sections. The cross-sectional properties of the replacement element are set to provide a good approximation of the minimal rotational support that the tapered element can provide to a connecting element. For more information on this limitation see Section 4.3.5.

The user shall become familiar with these limits and the decision on using the AutoN_{cr} Tool for a design project shall be made knowing the consequences of the assumptions and the methodology behind results. The examples in the following sections are developed to help getting familiar with the AutoN_{cr} Tool and its limits.

4. SINGLE-MEMBER EXAMPLES

Examples in this section feature a column supported by several types of constraints. The purpose of these examples is to show how the simplified model of the AutoN_{cr} Tool works and what its limits are. The first example is the simplest buckling problem - a simply supported column - that is followed by examples of increasing complexity.

These examples also serve as means of verification. Therefore, results of linear elastic buckling analysis are shown as reference and nonlinear static analyses are also performed on models with equivalent initial imperfections as per EC3-1-1 5.3.2 to show and compare results of more advanced analyses that are suitable for complex problems.

Several model parameters are kept identical for all examples. Members are assumed to be made of S235 grade structural steel (E_0 = 210 GPa; f_y = 23.5 kN/cm2; f_u = 36 kN/cm²). Members are modeled by beam elements using 20 finite elements for each member. Axial loads are introduced as centric concentrated forces. Flexural buckling around the weak axis and lateral torsional buckling of the investigated column is assumed to be prevented by properly installed lateral supports. Lateral torsional buckling of connecting members is also assumed to be prevented for simplicity. Eigen analyses refer to linear elastic buckling analyses. Auto refers to results of the AutoN_{cr} Tool.

Equivalent geometric imperfections are assumed to be identical to the buckled shape from the first buckling mode from linear elastic buckling analysis. Imperfection amplitudes are calculated with the methodology developed by Chladný and Štujberová (2013) that is also recommended in EC3-1-1 5.3.2(11). Nonlinear elastic analysis refers to a nonlinear static analysis on an imperfect geometry assuming linear elastic material response. Nonlinear inelastic analysis is identical in every respect, but the nonlinearity of material response is also considered. Following the recommendations in EC3-1-1 5.4.3(4) and EC3-1-5 (CEN, 2009b) C.6 a custom stress-strain diagram is defined to follow steel hardening under monotonic loading (*Figure 5*). All calculations have been performed in AxisVM.
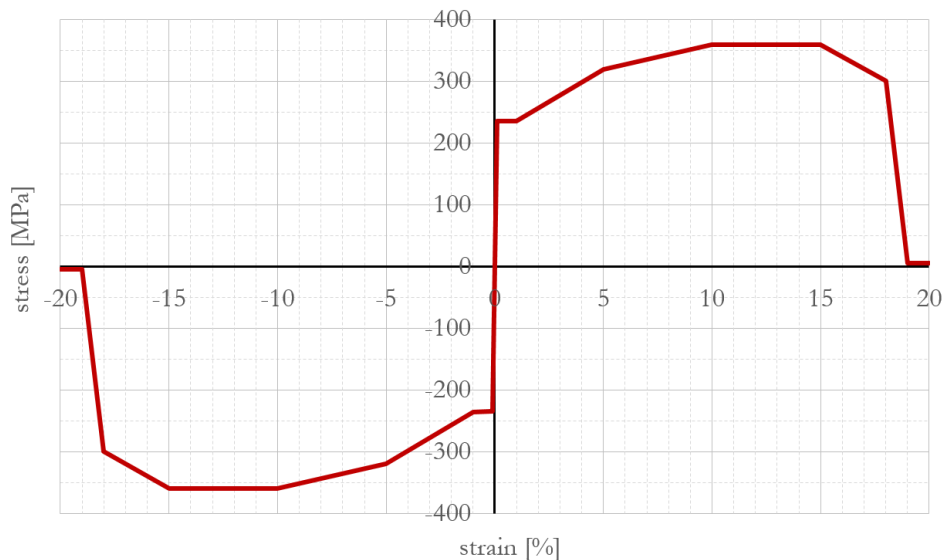

Figure 5 *Steel stress-strain diagram defined for nonlinear analysis*

Resistances and critical forces are calculated as follows:

- Eigenvalue analysis:

  The critical axial force (*eigen N_{cr}*) corresponds to the bifurcation point of a perfect system. The smallest eigenvalue of a linear elastic buckling analysis performed with 1 N axial load on the member under investigation is taken as N_{cr}. The buckling coefficient (*eigen K_y*) corresponding to a given N_{cr} is calculated as follows:

  $$K_y = \sqrt{\frac{\pi^2 EI}{N_{cr} L^2}}$$

  The flexural buckling resistance of the member (*eigen N_{bRd}*) is calculated by the Steel Designer Module in AxisVM as per EC3 using the eigen K_y as input.

- Automatic calculation:

  The critical axial force (*auto N_{cr}*) and the corresponding buckling coefficient (*auto K_y*) are calculated by the AutoN_{cr} tool in AxisVM using the methodology presented in Section 3. The flexural buckling resistance of the member (*auto N_{bRd}*) is calculated by the Steel Designer Module in AxisVM as per EC3 using the auto K_y as input.

- Nonlinear elastic analysis:

  The critical axial force (*nonlinear elastic N_{cr}*) is defined as the axial load at the inflection point of the axial load-maximum lateral displacement curve. The buckling resistance (*nonlinear elastic $N_{b,Rd}$*) is defined as an axial load value that leads to 100% utilization of the load bearing capacity of the member using strength checks (M-N-V in the Steel Designer Module in AxisVM) only.

- Nonlinear inelastic analysis:

  The buckling resistance (*nonlinear inelastic N_{bRd}*) is defined as the maximum point of the axial load-maximum lateral displacement curve (i.e. the maximum load bearing capacity of the numerical model).

4.1 Column with end supports only

The first set of examples investigate a single column with several options of nodal supports at its ends. The column has a length of 4 m, it is made of an HE200A section and loaded by a concentrated axial load at its top point.

4.1.1 Simply supported column

The first example is the basic flexural buckling problem: a simply supported column. *Figure 6a* summarizes the input data and shows the buckled shape of the column from linear elastic buckling analysis and nonlinear analysis. Note that the two shapes are practically identical.

The diagrams in *Figure 6b* show axial force - maximum lateral displacement (in the buckling plane) plots for several types of analyses. The lateral displacement for these plots is measured at the midpoint of the column. Important results are also summarized in *Table 1*.

Results of the AutoN_{cr} Tool perfectly match those of the eigenvalue analysis. The difference between the results of linear and nonlinear analyses is negligible.
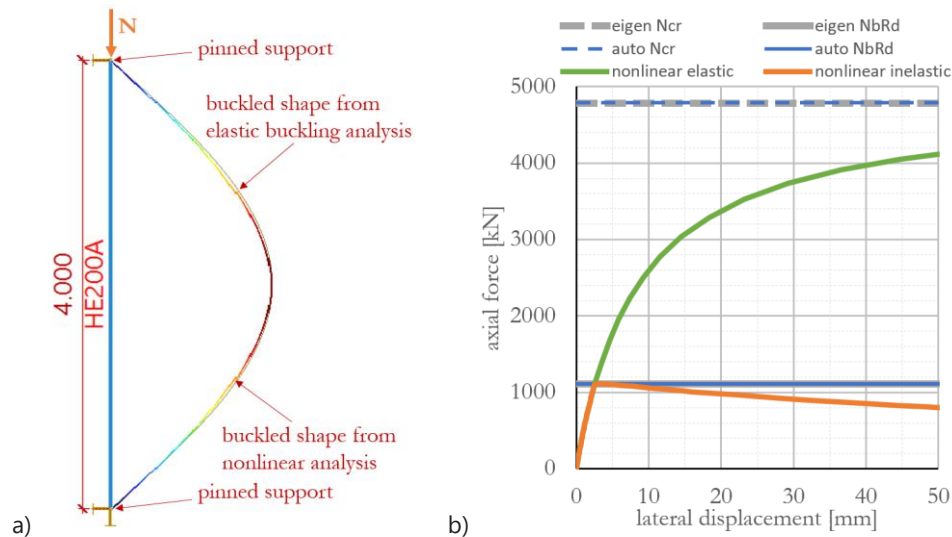
Figure 6 *Flexural buckling analysis of a simply supported column. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 1 *Result summary for flexural buckling analysis of a simply supported column. Forces are in kN.*

	K_y	N_{cr}	N_{bRd}
eigen	1.00	4784	1110
AutoN$_{cr}$	1.00	4784	1110
x nonlinear elastic	-	4809	1126
nonlinear inelastic	-	-	1116

4.1.2 Column in a sway frame with rigid supports

The second example introduces the sway property through a column that has no translational support at its top end (*Figure 8a*). As the EC3 background documentation (ECCS, 2006) explains in 4.3.2.1.2:

"The description "non-sway frame" applies to a frame when its response to in-plane horizontal forces is so stiff that it is acceptable to neglect any additional forces or moments arising from horizontal displacements of its storeys (so-called P-Δ effects). This means that the global second-order effects may be neglected. When the second-order effects are not negligible, the frame is said to be a "sway frame".

Note that sway and braced attributes of a frame structure describe different aspects of structural behavior. The sway attribute concerns sensitivity to second-order effects. The braced attribute concerns the existence of an adequately stiff bracing system. Thus, braced frames are not necessarily non-sway; the latter condition has to be verified independently of the bracing property.

EC3-1-1 5.2.1(3) suggests the following criteria to decide if second-order analysis is required:

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \quad \text{for elastic analysis}$$

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 15 \quad \text{for plastic analysis}$$

where F_{cr} is the critical buckling load corresponding to global instability, F_{Ed} is the design loading on the structure and α_{cr} is the critical load factor. If the influence of lateral torsional buckling on global instability is negligible, the load factor (n_{cr}) of the eigenvalue analysis (*Buckling* tab in AxisVM) is a good approximation of α_{cr}.

Second-order effects have significant influence on the elastic critical load and the buckling resistance of members in sway frames. The decision on the sway attribute of an element needs to be made by the user and communicated to the Auto*N_{cr}* Tool through the Steel Design Parameters dialog window (*Figure 7*). Note that buckling around the local y axis of the member occurs in the local x-z plane. Thus, the column in this example is set sway in the local x-z plane, but kept non-sway in the local y-z plane (because weak-axis flexural buckling is not considered here).

Input data and the buckled shape of the column are shown in *Figure 8a*. The buckled shape from nonlinear analysis and elastic buckling analyses are practically identical. Note that although the buckled shape in this case is different from that of the previous example, it corresponds to the same flexural buckling coefficient. Thus, the elastic critical load and the flexural buckling resistance is also identical to the results of the previous example (*Figure 8b*, *Table 2*). An important difference between the two examples is the maximum lateral displacement of the column that is 6.06 mm and 3.03 mm for the second and first example, respectively. (Maximum displacement in this example is measured at the top node of the column.)

Similar to the previous example, results of the Auto*N_{cr}* Tool perfectly match those of the eigenvalue analyses. The difference between the results of linear and nonlinear analyses is negligible.

Figure 7 *Setting the sway attribute of a design member in AxisVM.*

Table 2 *Result summary for flexural buckling analysis of a sway column with rigid supports. Forces are in kN.*

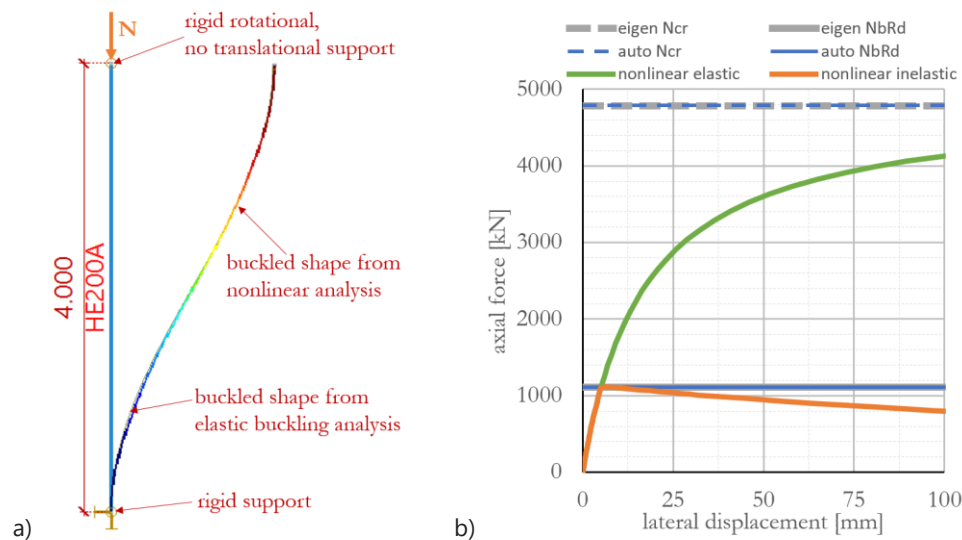|	K_y	N_{cr}	N_{bRd}
eigen	1.00	4784	1110
AutoN_{cr}	1.00	4784	1110
nonlinear elastic	-	4824	1127
nonlinear inelastic	-	-	1116

Figure 8 Flexural buckling analysis of a sway column with rigid supports. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

4.1.3 Cantilever with rigid support

The third example presents a cantilever. This member shall be considered sway, because the lack of translational support at its top node makes it sensitive to second-order effects. Input data and the buckled shape are shown in *Figure 9a*. *Figure 9b* and *Table 3* presents the results of different analyses. All analysis options provide results with minimal error compared to the reference eigenvalue-based solution.
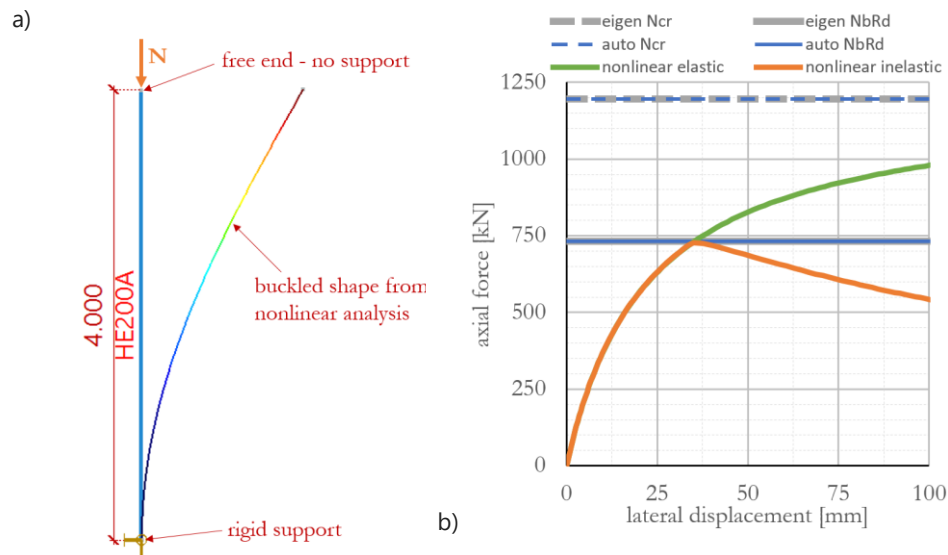

Figure 9 Flexural buckling analysis of a cantilever. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

Table 3 Result summary for flexural buckling analysis of a cantilever. Forces are in kN.

	K_y	N_{cr}	N_{bRd}
eigen	2.00	1196	732.6
AutoN$_{cr}$	2.00	1196	732.6
nonlinear elastic	-	1198	745.0
nonlinear inelastic	-	-	727.7

4.1.4 Cantilever with semi-rigid support

The first three examples were textbook cases that any practicing engineer can solve without the aid of a computer. This is the first model that goes beyond those special cases by the assumption of a semi-rigid support at the bottom of a cantilever. The previous example shows results with a perfectly rigid support. If free rotation is allowed at the base of the column, the system is unstable: the corresponding critical axial force is zero. Results with a semi-rigid support shall be in between these two special cases.

Support rigidity is defined as $R_{YY} = 2EI/L$, where *E*, *I* and *L* are the initial stiffness of the steel material, the strong-axis moment of inertia of the column's cross section and the length of the column, respectively. Note that due to the finite rigidity of the support, the tangent of the buckled shape at the base of the column is not vertical in *Figure 10a* (compare to *Figure 9a*).

The finite rigidity of the support is also apparent in the results of the analyses (*Figure 10b*, *Table 4*). The elastic critical axial force is 47%, while the buckling resistance is reduced to 59% when compared to the previous example. Note that a flexural buckling coefficient of 2.92 corresponds to this phenomenon.

Although the variance of results in this example is larger than in previous ones, it is still within the acceptable range considering the inherent uncertainty in stability analyses.
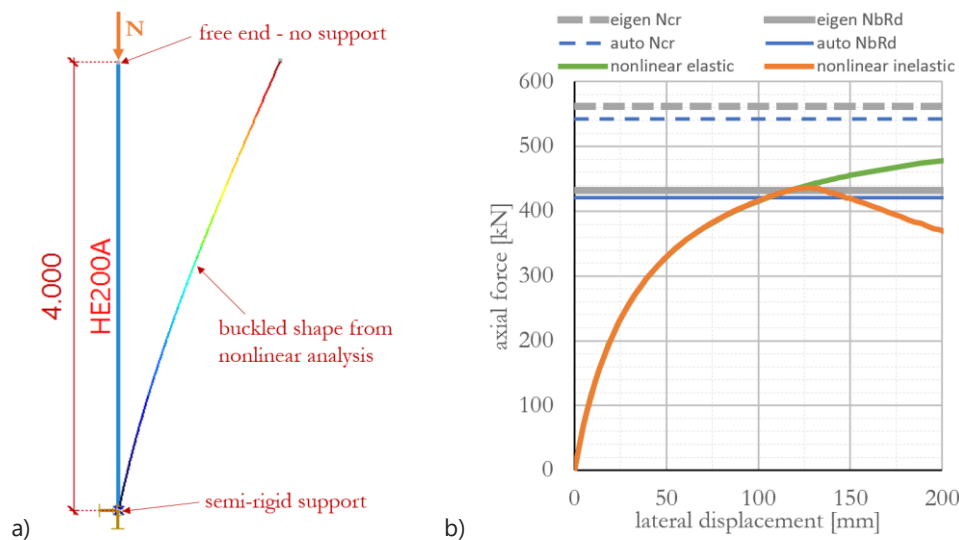

Figure 10 Flexural buckling analysis of a cantilever with semi-rigid supports. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

Table 4 Result summary for flexural buckling analysis of a cantilever with semi-rigid supports. Forces are in kN.

	K_y	N_{cr}	N_{bRd}
eigen	2.92	562	432.2
AutoN$_{cr}$	2.97	542	421.3
nonlinear elastic	-	558	444.7
nonlinear inelastic	-	-	435.8

4.1.5 Accuracy of the AutoNcr Tool for single columns

The previous example has shown that it is more challenging to determine the elastic critical axial force for members with semi-rigid supports than for the special cases with either perfectly rigid or free constraints. This final part of the first set of examples shows how the accuracy of the AutoN_{cr} Tool is affected by the semi-rigid nature of supports. Note that good accuracy of such a tool at this point is a must for practical applicability, because support stiffness from connecting beams and columns often falls within the semi-rigid range.

The case of a simply supported column from the first example is enhanced first. The diagrams in *Figure 11* show the elastic critical axial force (N_{cr}), the buckling resistance (N_{bRd}) and the flexural buckling coefficient (K_y) as a function of the stiffness of the top support. Top support stiffness is given relative to the bending stiffness of a reference column that is rigidly supported at its bottom node and pinned at the top node ($4EI/L$).

Until the top support stiffness exceeds 0.01 times the reference column stiffness, results are similar to those of the first example: K_y = 1.0, N_{cr} = 4787 kN, and N_{bRd} = 1110 kN. As the stiffness of the top support is further increased, N_{cr} and N_{bRd} increases, while K_y decreases gradually. If the top support stiffness exceeds 100 times the reference column stiffness, results are very close to the special rigid-pinned case with K_y = 0.699, N_{cr} = 9792 kN, and N_{bRd} = 1192 kN.

The AutoN_{cr} Tool provides perfect results for the special cases, but more importantly it also provides highly accurate results for the cases with semi-rigid supports (compare the continuous lines with the dashed lines in *Figure 11*). The maximum error is 0.90%, 1.80% and 0.22% for K_y, N_{cr}, and N_{bRd}, respectively.
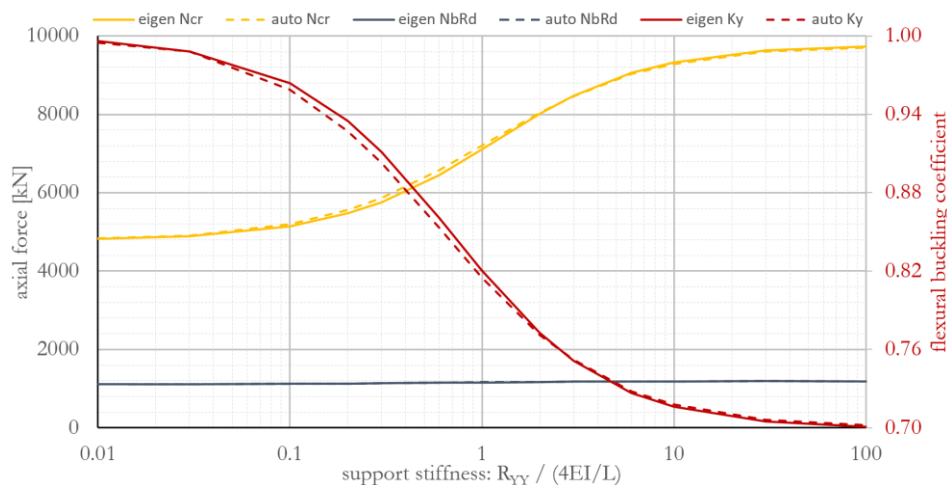

Figure 11 Flexural buckling attributes of a column with a pinned bottom and a semi-rigid top support.
Continuous lines show reference results based on eigenvalue analysis; dashed lines correspond to results of the AutoN$_{cr}$ Tool.

The cantilever from the third example is investigated in *Figure 12*. Stiffness of the bottom support is modified from a practically pinned to a practically rigid setup. The practically rigid setup corresponds to the third example in Section 4.1.3. The fourth example is at 0.5 in the horizontal axis of *Figure 12*, because the support stiffness is $2EI/L$ in that case. Note how the difference between the elastic critical axial load and the flexural buckling resistance diminishes with the reduction of support stiffness.

The AutoN_{cr} Tool provides results with good accuracy even for low support stiffnesses and flexural buckling coefficients beyond 2.0. The maximum error in the cases shown in *Figure 12* is 1.80%, 3.70% and 3.10% for K_y, N_{cr}, and N_{bRd}, respectively. Considering the uncertainty in the stability analysis of such cantilevers, these results are deemed sufficiently accurate for practical application.
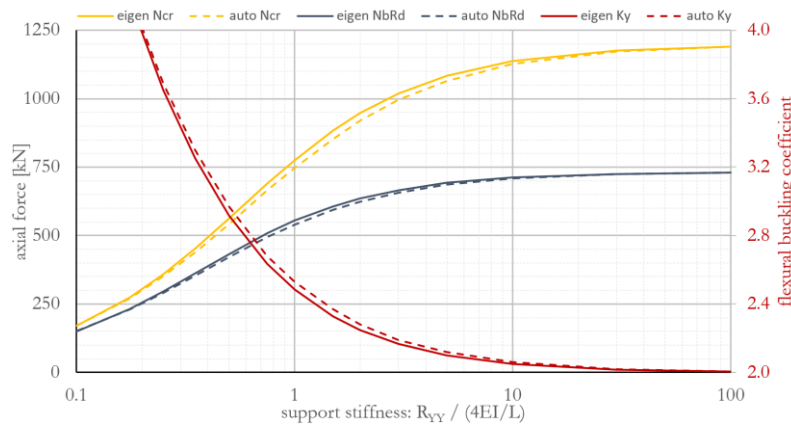

Figure 12 *Flexural buckling attributes of a cantilever with a semi-rigid bottom support.*
Continuous lines show reference results based on eigenvalue analysis; dashed lines correspond to results of the AutoN$_{cr}$ Tool.

4.2 Column with multiple supports

The following examples demonstrate the applicability of the AutoN_{cr} Tool for the analysis of a column with several supports along its length. The general parameters (e.g. column cross section, material characteristics, analysis settings) are identical to those of the previous examples.

4.2.1 Support at midpoint under uniform compression

The first example investigates the behavior of a simply supported column with a pinned support at its midpoint. *Figure 13a* summarizes the input data and shows the buckled shape of the column. Note that the column in this example is made up from two segments, each being identical to the simply supported column in Section 4.1.1. The important difference between the two examples is that there is a connection between the two segments and through this connection their behavior is interrelated (i.e. N_{cr} of each segment considered independently affects N_{cr} of the system). This makes the problem in *Figure 13* more difficult to handle than the simple cases in Section 4.1.

Results of different analyses are summarized in the diagrams of *Figure 13b* and in *Table 5*. Because the top and bottom parts of the column are identical, their stability failure is characterized by the same N_{cr} value. Note that this N_{cr} is identical to that of the example in Section 4.1.1 as well. The AutoN_{cr} Tool performs very well in this special case and the other analysis options also provide results with minimal error.

Table 5 Result summary for flexural buckling analysis of a column supported at its midpoint. Forces are in kN.

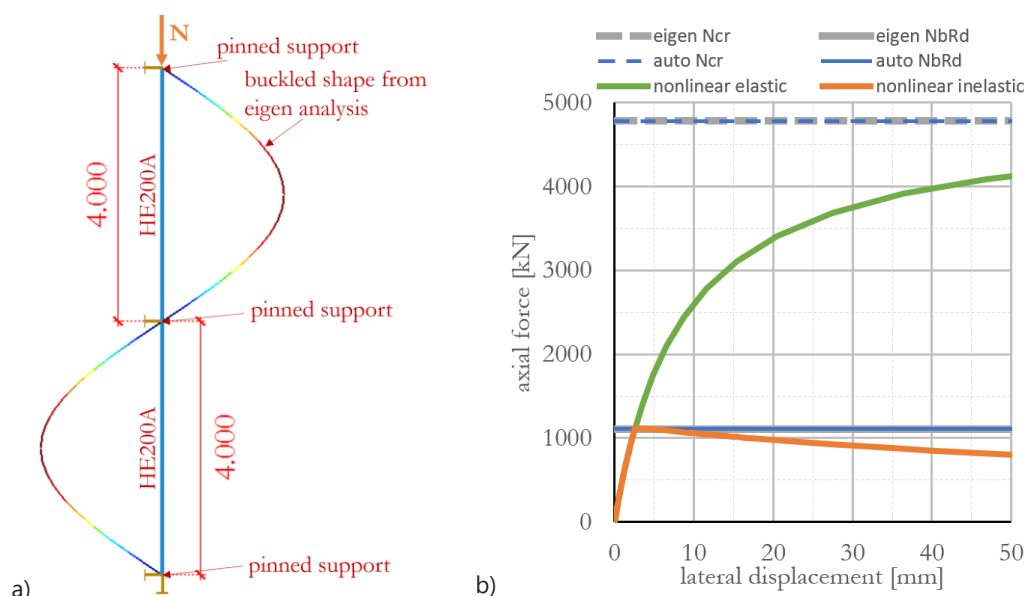|	K_y	N_{cr}	N_{bRd}
eigen	1.00	4784	1111
AutoN$_{cr}$	1.00	4784	1111
nonlinear elastic	-	4813	1128
nonlinear inelastic	-	-	1116

Figure 13 Flexural buckling analysis of a column supported at its midpoint. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

4.2.2 Support at midpoint under piecewise uniform compression

This example highlights the importance of the normal force distribution along the element. The inputs are identical to the previous example except for the axial force: half of the force is introduced at the top of the column, while another half acts at the middle support (*Figure 14a*). Thus, the top and bottom parts experience 0.5 *N* and *N* normal force, respectively. Because N_{cr} is defined as the product of a critical load factor (the smallest eigenvalue) and the initial load, N_{cr} of the top part will be half of N_{cr} of the bottom part. This yields a higher flexural buckling coefficient and lower flexural buckling resistance for the top column.

These results might be surprising at first, but a different explanation of the phenomenon reveals the reason behind them. Let us imagine the search for N_{cr} as an experiment where the axial load on the structure is gradually increased until stability failure occurs. In such an experiment we would see that transversal displacements of the bottom part are larger (assuming identical imperfection on both parts), because the axial load and the corresponding second order bending moment is always larger in the bottom part. However, the bottom part cannot be simplified to the case of a simply supported column. Because the top part experiences considerably less compression, it can provide rotational support to the bottom part at their connection point at the middle of the column. Thus, the bottom part will fail at a higher compression force and in return, the top part will have a smaller N_{cr} than a simply supported column. This leads to an equilibrium where the two parts buckle simultaneously.
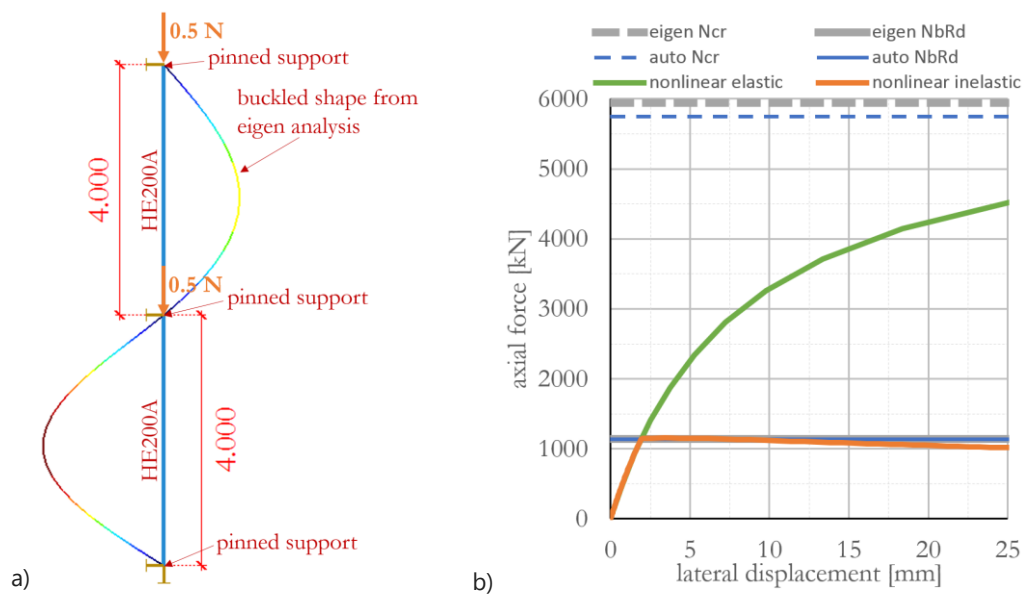
Figure 14 *Flexural buckling analysis of a column under piecewise uniform compression. a) Input and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 6 *Result summary for flexural buckling analysis of a column under piecewise uniform compression. Forces are in kN. (N_{bRd} values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)*

		K_y	N_{cr}	N_{bRd}
TOP	**eigen**	1.27	2971	1025
	AutoN$_{cr}$	1.29	2879	1018
	nonlinear elastic	-	2996	(576.9)
	nonlinear inelastic	-	-	(577.1)
BOTTOM	**eigen**	0.90	5943	1140
	AutoN$_{cr}$	0.91	5752	1136
	nonlinear elastic	-	5991	1154
	nonlinear inelastic	-	-	1154

If the stability failure of a structural member is considerably influenced by the axial load on another member, the axial load shall always be taken into consideration by the AutoN_{cr} Tool. This shall be specified by the user through the Steel Design Parameters dialog window (*Figure 15*). This leads to an individual calculation based on the internal force distribution that corresponds to a specific loading scenario. Given a large number of load combinations and a complex structural model, the number of calculations and the required computation time can become significant.

The user can decide to use an approximate solution to speed up the calculation. The approximate solution assumes uniform compression force along the element under consideration and neglects compression effects in every other element. This simplification will reduce computation time, but it is important to understand its limitations and use it only when applicable. In case of this example, the approximate approach would not identify the difference between the behavior of the top and bottom parts of the column and it would yield results that are identical to those of the previous example. Thus, it should not be used in such cases.

Figure 14b and *Table 6* present the results of different analyses. Note how the results changed compared to those in *Table 5*. Results of alternative analyses are in good agreement with the target results of the eigen analysis. Nonlinear analyses confirm that the top part has a supporting role in the structure: although its N_{cr} is lower when compared to a simply supported case, its load at the time of global stability failure (577 kN) is still less than 60 % of its flexural buckling resistance. The critical component is the bottom part of the column.
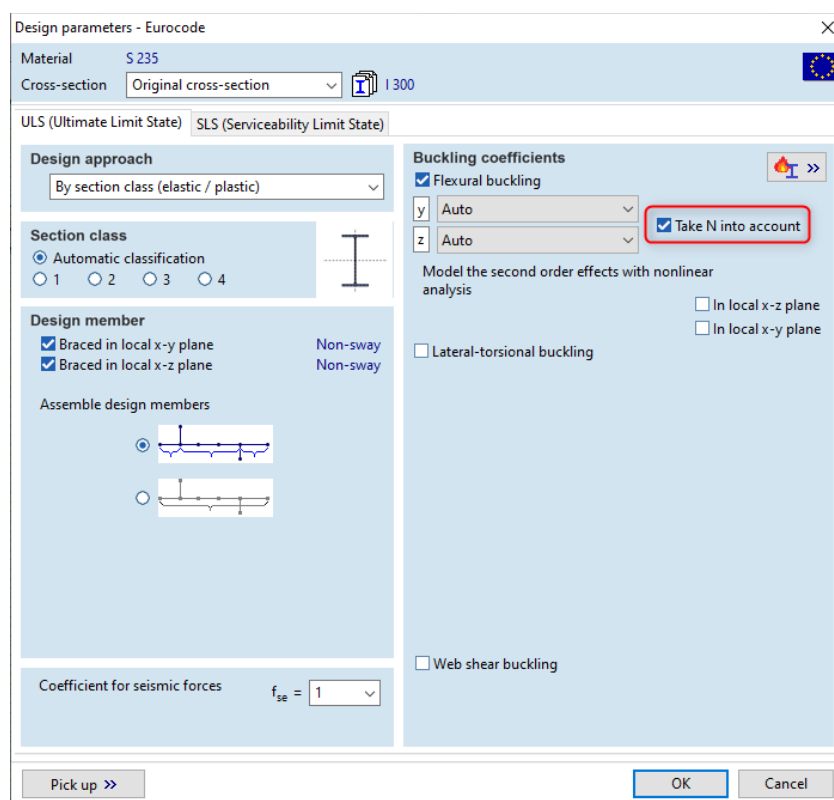

Figure 15 *Specify if the AutoN_{cr} Tool shall consider load-specific normal forces in AxisVM.*

4.2.3 Supports at several points

This example shows a structure with four supports: the top and middle segments are half the length of the bottom segment (*Figure 16a*). The normal force is uniformly distributed in this case. Because the internal forces are identical in all segments, the N_{cr} values are also equal for all of them. Note however, that the flexural buckling coefficients are heavily affected by the interaction between the segments. Similarly to the previous example, the bottom segment is supported by those above it (compare $K_{y,bottom} = 0.8$ to $K_{y,top} = 1.6$ in *Table 6*). An important difference here is in the reason for this condition: the top segments are shorter thus they would buckle at a much higher N_{cr} were they simply supported. Their connection to an element with less flexural rigidity (the bottom segment) makes them more susceptible to buckling and reduces the corresponding N_{cr} value.

Figure 16b and *Table 6* present the results of different analyses. All analysis options provide results with minimal error compared to the reference eigen solution. Note that due to the uniform normal force distribution, the AutoN_{cr} Tool in this example can be used without the consideration of actual internal forces in the columns (*Figure 15*).
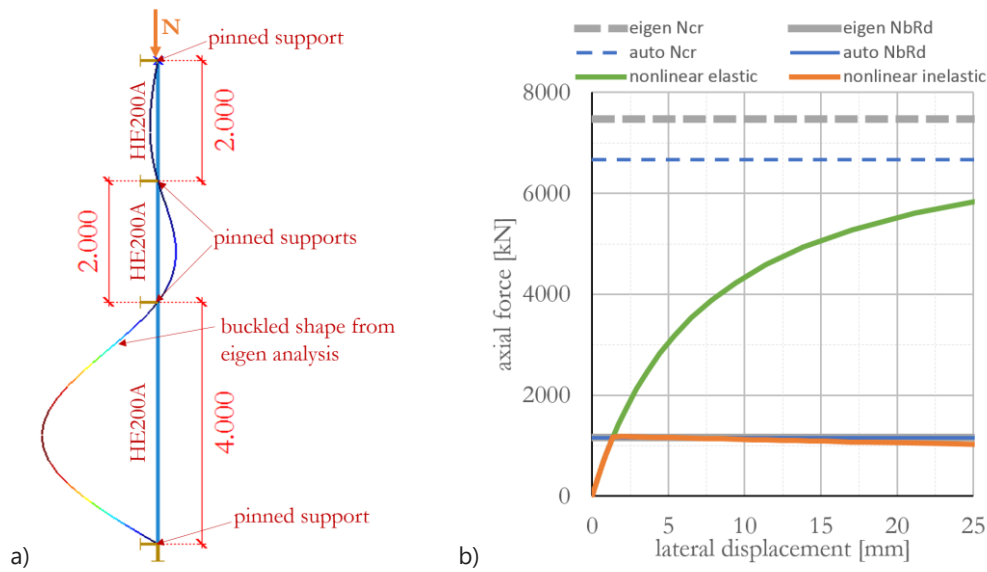
Figure 16 *Flexural buckling analysis of a column supported at several points. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 7 *Result summary for flexural buckling analysis of a column supported at several points. Forces are in kN.*

		K_y	N_{cr}	N_{bRd}
TOP, MIDDLE	**eigen**	1.60	7474	1166
	AutoN$_{cr}$	1.67	6668	1154
	nonlinear elastic	-	7612	1177
	nonlinear inelastic	-	-	1180
BOTTOM	**eigen**	0.80	7474	1166
	AutoN$_{cr}$	0.85	6668	1154
	nonlinear elastic	-	7612	1177
	nonlinear inelastic	-	-	1180

4.2.4 Complex set of supports and piecewise uniform compression

The last illustrative example in this section shows a structure closer to everyday practice. The column has four semi-rigid rotational supports that represent a foundation and connecting beams on three stories. Support rotational stiffnesses are 5x, 2x, 2x, and 1/8x the flexural rigidity of the column ($4EI/L_{bottom}$) for the foundations, and the three stories, respectively (*Figure 17a*). Axial load is introduced in 0.33 *N* increments. The resulting piecewise uniform load distribution for the column represents the axial loading of typical frame columns. Note that the column has uniform cross-section, which is not typical. Modelling non-uniform cross-sections is discussed among the limitations in Section 4.2.6. The influence of second-order effects is considered significant. Therefore, only the base provides translational support and the column is considered to be in a sway-frame (see Section 4.1.2 for details on the sway property)

Figure 17b and *Table 8* summarize the results. The interaction between column segments is similar to the previous examples. The bottom segment is supported by the middle and top segments. Nonlinear analysis results confirm that the bottom segment is the critical component of the structure. All analysis options provide

results with reasonable accuracy for all components. This confirms that flexural buckling resistance of columns with such complex supporting and loading conditions can be approximated with the AutoN_{cr} Tool with sufficient accuracy.
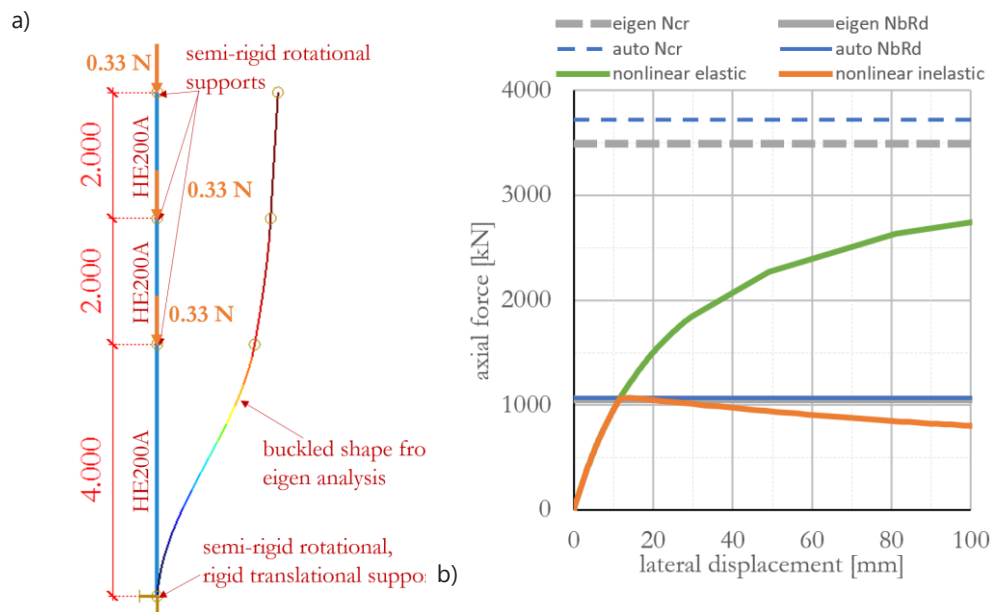

Figure 17 *Flexural buckling analysis of a column with a complex set of supports and piecewise uniform compression. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 8 *Result summary for flexural buckling analysis of a column with a complex set of supports and piecewise uniform compression. Forces are in kN. (N_{bRd} values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)*

		K_y	N_{cr}	N_{bRd}
TOP	**eigen**	4.06	1163	721.2
	AutoN$_{cr}$	3.94	1234	745.2
	nonlinear elastic	-	1169	(359.1)
	nonlinear inelastic	-	-	(355.8)
MIDDLE	**eigen**	2.87	2326	964.3
	AutoN$_{cr}$	2.78	2478	981.2
	nonlinear elastic	-	2338	(718.2)
	nonlinear inelastic	-	-	(711.7)
BOTTOM	**eigen**	1.17	3493	1058
	AutoN$_{cr}$	1.13	3720	1070
	nonlinear elastic	-	3508	1078
	nonlinear inelastic	-	-	1069

4.2.5 Accuracy: piecewise uniform compression and various relative lengths

In this section accuracy of the AutoN_{cr} Tool is demonstrated through two case studies. The first analysis investigates the effect of normal force distribution along the length of a column supported by three pinned supports. The diagrams in *Figure 18* compare N_{cr}, N_{bRd} and K_y corresponding to the bottom column segment

from eigen analysis to the results using the AutoN_{cr} Tool. Note that examples in Sections 4.2.1 and 4.2.2 are special cases of this analysis with an N_{top}/N_{bottom} value of 1.0 and 0.5, respectively.

The two extreme cases in Figure 18 are a column with no axial load on its top or bottom segment. Both cases are equivalent to a simply supported segment with additional rotational rigidity at one of its supports.

The figure focuses on the flexural buckling properties of the bottom segment. As the axial load ratio increases beyond 1.0 and the bottom segment receives less load, its role as a supporting member becomes more significant. As the axial load ratio in *Figure 18* approaches infinity, N_{cr} and N_{bRd} of the bottom segment approach zero. Because the compression force (N_{Ed}) approaches zero at a faster rate than N_{bRd}, the bottom segment will not become the critical component at the right end of the axial load ratio spectrum. This is an important observation that will be revisited at Section 4.3.

The AutoN_{cr} Tool approximates the eigen results with small errors for the whole range of axial load ratios. The maximum errors are 1.55%, 3.20%, and 1.50% for K_y, N_{cr}, and N_{bRd}, respectively.
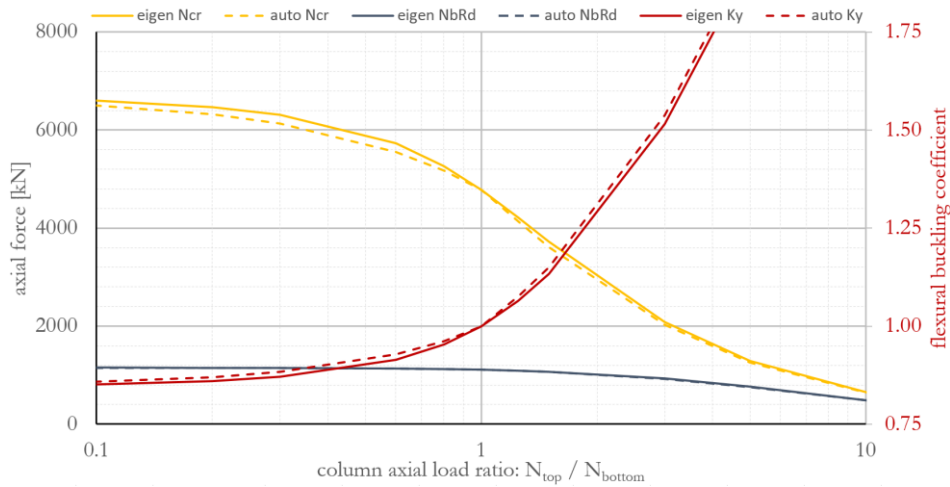

Figure 18 Flexural buckling attributes of a column with three supports and piecewise uniform axial load.
Continuous lines show reference results based on eigenvalue analysis; dashed lines correspond to results of the AutoN$_{cr}$ Tool.
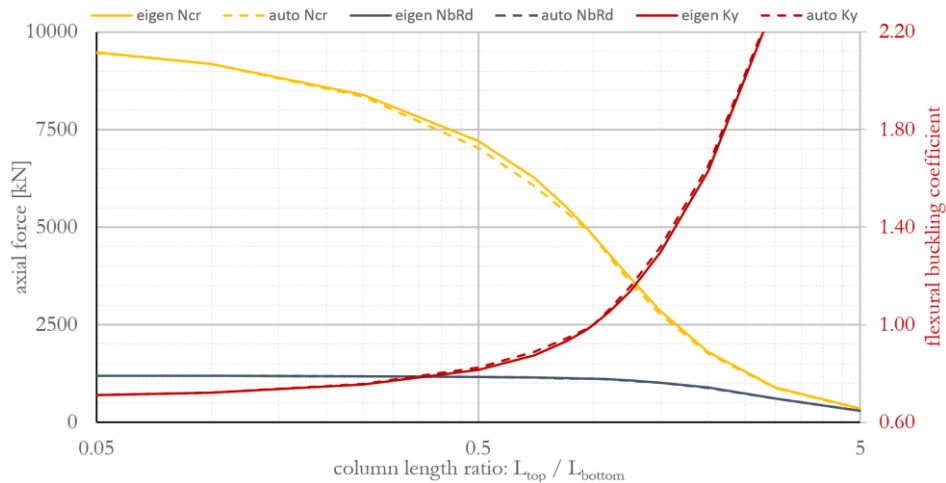

Figure 19 Flexural buckling attributes of a column with three supports and various top segment length.
Continuous lines show reference results based on eigenvalue analysis; dashed lines correspond to results of the AutoN$_{cr}$ Tool.

The second analysis in *Figure 19* investigates the influence of relative rigidity of column segments on the buckling parameters. It is based on the simple column configuration in Section 4.2.1, but the length of the top segment is a variable in the range of 20 cm to 20 m. Note that the general tendencies are similar to the previous example in *Figure 18*, but the two cases are not identical. As the length of the top segment approaches zero (left part of the figure) its flexural rigidity approaches infinity. Thus, the behavior of the

bottom segment becomes similar to a column with a fixed and a pinned support characterized by a flexural buckling coefficient of 0.699.

As the length of the top segment is increased, its flexural rigidity becomes similar to that of the bottom segment. They are identical at a column length ratio of 1.0. After that point the bottom segment supports the top one. This explains why K_y of the bottom segment exceeds 1.0 at that part of the diagram. As the length of the top segment approaches infinity, N_{cr} approaches zero. Note that although N_{bRd} of the bottom segment approaches zero, the corresponding N_{Ed} is always smaller. Thus, for such cases, the top segment will always be the critical component.

The AutoN_{cr} Tool approximates the eigen results with small error for the whole range of column length ratios. The maximum errors are 1.70%, 3.50%, and 1.05% for K_y, N_{cr}, and N_{bRd}, respectively.

4.2.6 Applicability limits: Non-uniform axial load

This example highlights the first limit to the application of the AutoN_{cr} Tool. Although the tool performs well for structures with piecewise uniform axial load on its members, it is not designed to approximate the consequences of a more general load distribution. The column in *Figure 20a* is loaded at the middle of its bottom segment. This results in non-uniform compression force distribution in the bottom segment that makes the calculation of flexural buckling parameters more challenging.

The eigen analysis provides reference results (*Table 9*). Note that the top segment has infinite flexural buckling length and zero N_{cr}, because its N_{Ed} is zero. (The problem is similar to the right extreme case in *Figure 18*.) Results in the AutoN_{cr} Tool are limited at $K_y = 10.0$ to make sure that a large K_y does not interfere with other parts of the calculation. Results with $K_y = 10.0$ are deemed similar to results with an infinite K_y from a practical viewpoint, since such results occur only if N_{Ed} is very small or the member is under tension.
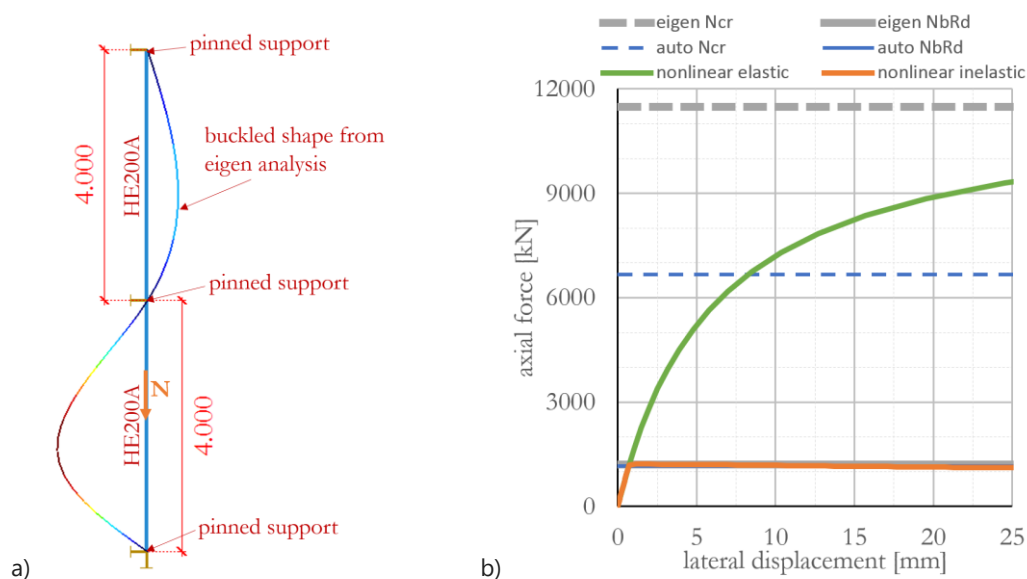

Figure 20 Flexural buckling analysis of a column with non-uniform axial load. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

Limits of the AutoN_{cr} Tool are apparent at the bottom segment. The normal force is introduced at the middle of the segment, but the tool does not take that into consideration. It always uses the maximum normal force along a segment and assumes a uniform distribution. Thus, it determines the flexural buckling parameters for a column that is loaded at its midpoint (at the top of the bottom segment). This approach always provides conservative results. Note that although the differences in K_y and N_{cr} are significant, N_{bRd} is only slightly underestimated by this conservative approach. Nevertheless, it is important to recognize these loading situations and only use the AutoNcr Tool if the conservativism in the results is acceptable.

Table 9 *Result summary for flexural buckling analysis of a column with non-uniform axial load. Forces are in kN.*

		K_y	N_{cr}	N_{bRd}
TOP	**eigen**	∞	0	0
	AutoN$_{cr}$	10.0	47.8	44.9
	nonlinear elastic	-	0	(0.0)
	nonlinear inelastic	-	-	(0.0)
BOTTOM	**eigen**	0.646	11478	1205
	AutoN$_{cr}$	0.847	6668	1154
	nonlinear elastic	-	11429	1187
	nonlinear inelastic	-	-	1215

4.2.7 Applicability limits: Non-uniform column section

The second example on applicability limits concerns non-uniform column cross-sections. There are two examples in this part: the first shows a situation, where the AutoN_{cr} Tool provides appropriate results, while the second shows a configuration where it does not.

The first example is similar to the complex case in Section 4.2.4, but the supports are pinned and the column is non-sway. The top and middle segments are made of HE140A sections (*Figure 21a*). Because the AutoN_{cr} Tool assumes that structural members have uniform sections, it separates the column in this example to two members. Although these members influence each other through the connection at the top of the bottom segment in the AutoN_{cr} model, their common behavior is only approximated using the approach recommended by ECCS (see Sections 3 and 4.3 for details). Consequently, the influence of the top segment on the behavior of the bottom segment is considered with less accuracy than it is for a column with uniform cross-section.

Figure 21b and *Table 10* show that results of the AutoN_{cr} Tool are sufficiently accurate for all segments in this configuration. It successfully identifies that failure is expected in the middle segment - this is also confirmed by nonlinear analyses.

Table 10 Result summary for flexural buckling analysis of a column with non-uniform cross-section - first example. Forces are in kN. (N$_{bRd}$ values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)

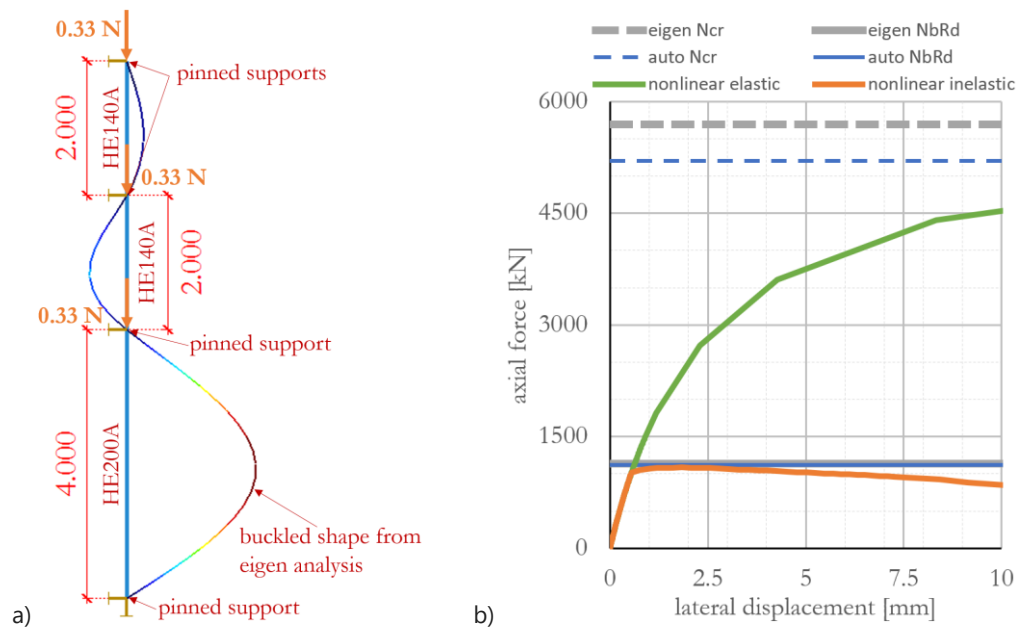|		K_y	N_{cr}	N_{bRd}
TOP	**eigen**	1.68	1896	609.0
	AutoN$_{cr}$	1.61	2063	618.7
	nonlinear elastic	-	1918	(339.3)
	nonlinear inelastic	-	-	(361.7)
MIDDLE	**eigen**	1.19	3792	671.5
	AutoN$_{cr}$	1.14	4120	677.0
	nonlinear elastic	-	3835	678.5
	nonlinear inelastic	-	-	723.4
BOTTOM	**eigen**	0.92	5693	1135
	AutoN$_{cr}$	0.96	5202	1122
	nonlinear elastic	-	5759	(1019)
	nonlinear inelastic	-	-	(1086)

Figure 21 Flexural buckling analysis of a column with non-uniform cross-section - first example.
a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.

The second example uses the structural configuration in *Figure 22a*. Note that half of the bottom segment in this case is made of HE140A cross-section, while the other half remains HE200A. This slight modification leads to a very different behavior when compared to the first example (see the difference in the buckled shape for instance). N_{cr} of the column is reduced significantly and nonlinear analyses results confirm that the critical section moves to the top part of the bottom segment (Table 11).

Currently, this behavior cannot be properly approximated by the AutoN_{cr} Tool. The tool identifies two individual members in this example that are connected at the midpoint of the bottom segment. Because this is a non-sway column, the tool assumes a translational support at all connections - including the one between the two members. This mistake causes the error in flexural buckling parameters. Note that the error is not on the conservative side.

These results highlight the importance of being familiar with the application limits of the AutoN_{cr} Tool and using it only when it is expected to provide sufficiently accurate results. Because this second example does not occur frequently in practical application, the corresponding limitation in the AutoN_{cr} Tool is considered an acceptable tradeoff for faster and more efficient calculation in other, more common cases.
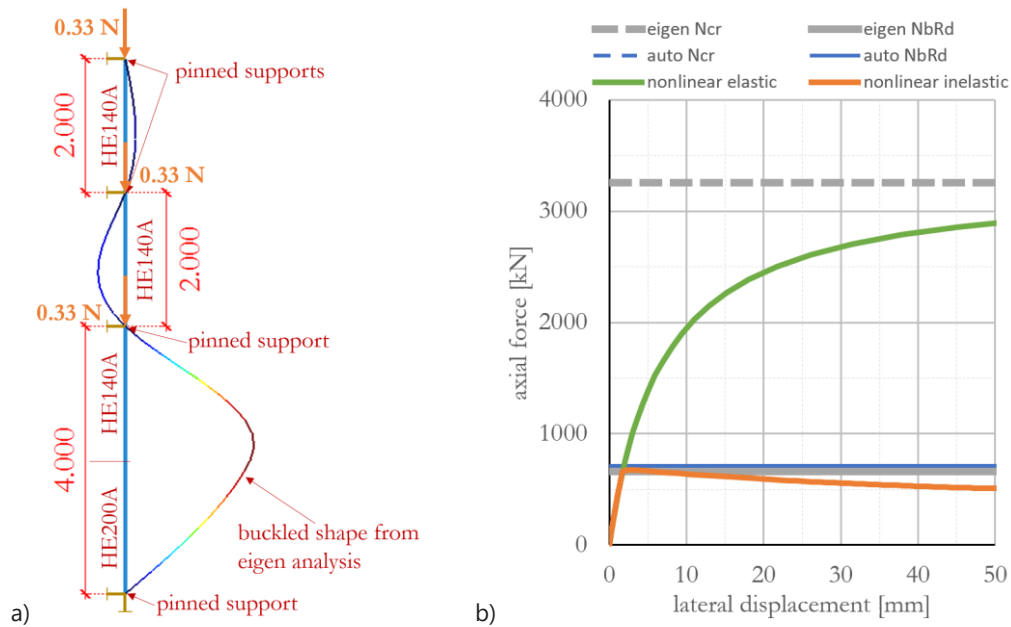

Figure 22 *Flexural buckling analysis of a column with non-uniform cross-section - second example. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 11 Result summary for flexural buckling analysis of a column with non-uniform cross-section - second example. Forces are in kN. (N_{bRd} values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)

|		K_y	N_{cr}	N_{bRd}
TOP	eigen	2.22	1083	523.2
	AutoN$_{cr}$	1.44	2575	641.1
	nonlinear elastic	-		(223.3)
	nonlinear inelastic	-	-	(224.0)
MIDDLE	eigen	1.57	2166	624.0
	AutoN$_{cr}$	1.02	5156	690.4
	nonlinear elastic	-		(446.6)
	nonlinear inelastic	-	-	(447.9)
BOTTOM HE140A	eigen	1.28	3253	660.5
	AutoN$_{cr}$	0.83	7735	709.4
	nonlinear elastic	-		670.6
	nonlinear inelastic	-	-	672.6
BOTTOM HE200A	eigen	2.43	3253	1043.9
	AutoN$_{cr}$	1.17	3918	1219.6
	nonlinear elastic	-		(670.6)
	nonlinear inelastic	-	-	(672.6)

4.3 Column supported by other members

The following examples demonstrate the applicability of the AutoN$_{cr}$ Tool for the analysis of a column that is supported by other structural members. The parameters (e.g. column cross section, material characteristics, analysis settings) are identical to those of the previous examples.

4.3.1 Simple beam as a support

The first example shows a basic configuration: a pinned, non-sway column supported by a single beam. We shall find the results in *Figure 11*, where a single column with variable semi-rigid supports was investigated. The rigidity of the top support in this case depends on the length, supporting conditions, and cross-section of the beam.

The structural layout is shown in *Figure 23a*, while the results are summarized in *Figure 23b* and *Table 12*. Column behavior is properly approximated by the AutoN$_{cr}$ Tool. Because the beam does not experience compression, the K_y = 10.0 limit of the tool governs the automatically calculated results. Similar to the example in Section 4.2.6, the beam is not expected to fail due to flexural buckling, and the error in its parameters are not important from a practical viewpoint.
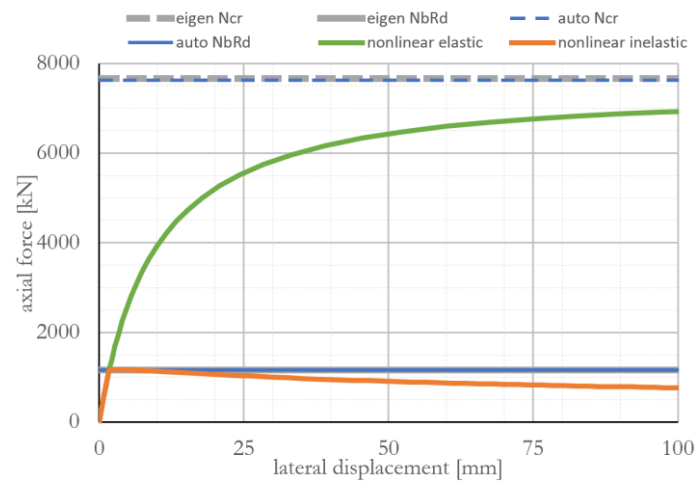
a)
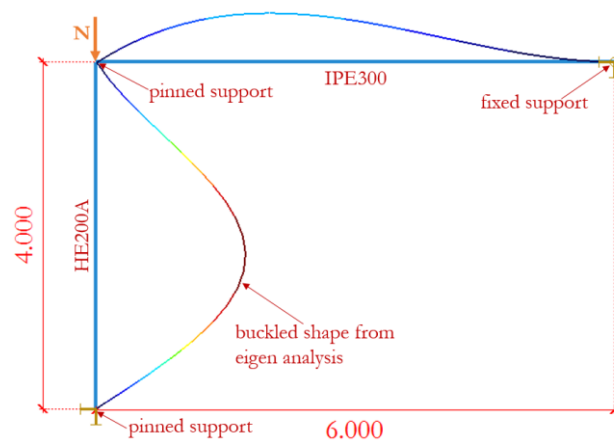
b)

Figure 23 *Flexural buckling analysis of a column supported by a simple beam. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*
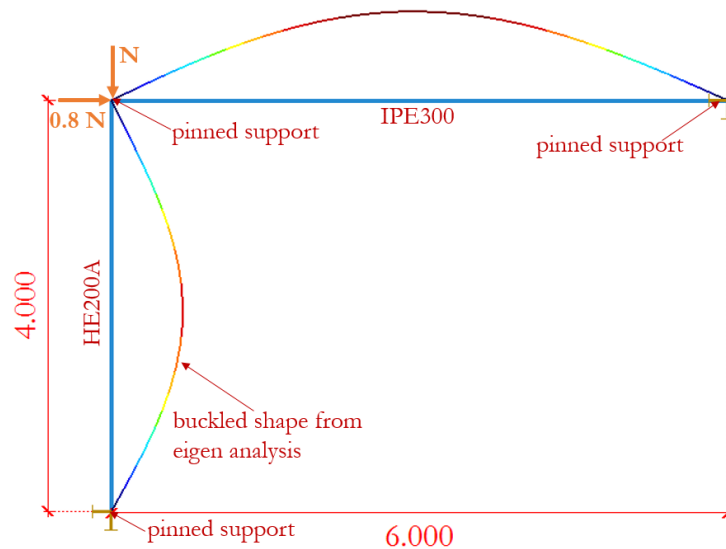
Table 12 *Result summary for flexural buckling analysis of a column supported by a simple beam. Forces are in kN. (N_{bRd} values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)*

		K_y	N_{cr}	N_{bRd}
BEAM	**eigen**	∞	0	0
	AutoN$_{cr}$	10.0	48.1	46.2
	nonlinear elastic	-	0	(0.0)
	nonlinear inelastic	-	-	(0.0)
COLUMN	**eigen**	0.79	7660	1169
	AutoN$_{cr}$	0.79	7627	1168
	nonlinear elastic	-	7095	1167
	nonlinear inelastic	-	-	1172

4.3.2 Supporting beam under compression

This example highlights the effect of compression force on the available supporting stiffness of a connecting member. The structural layout is similar to the previous case, but the other end of the beam is pinned in this example. Besides the vertical load, a horizontal concentrated force is introduced. The magnitude of the horizontal force is 80% of the vertical one (Figure 24a).
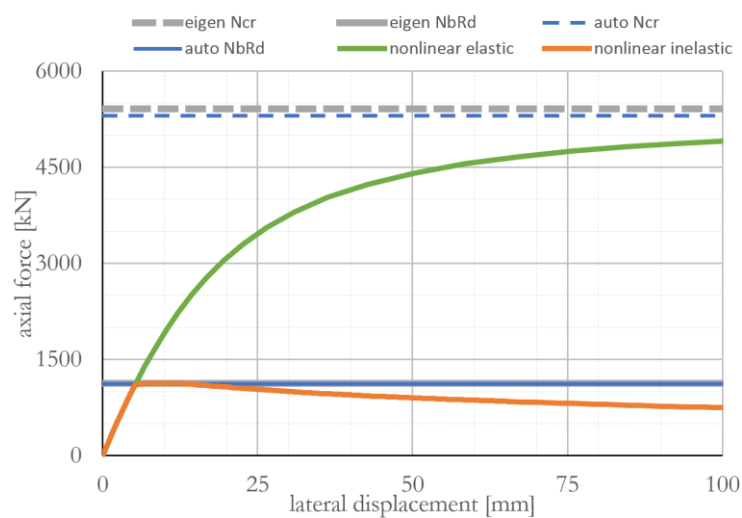
a)

b)

Figure 24 *Flexural buckling analysis of a column supported by a beam under compression. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 13 *Result summary for flexural buckling analysis of a column supported by a beam under compression. Forces are in kN. (N$_{bRd}$ values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)*

		K_y	N_{cr}	N_{bRd}
BEAM	**eigen**	1.06	4326	1152
	AutoN$_{cr}$	1.04	4475	1156
	nonlinear elastic	-	4127	(890.0)
	nonlinear inelastic	-	-	(901.5)
COLUMN	**eigen**	0.94	5408	1128
	AutoN$_{cr}$	0.95	5301	1125
	nonlinear elastic	-	5159	1113
	nonlinear inelastic	-	-	1127

Figure 24b and *Table 13* summarize the results. The compression force in the beam reduces its available supporting stiffness and leads to a higher K_y value and lower N_{cr} for the column (The same configuration without the axial force in the beam would lead to K_y = 0.81 and N_{cr} = 7269 kN for the column). Note that the same logic can be used for the beam: Although it is pinned at one end and supported by a column at the other, given the loading conditions in *Figure 24a*, its flexural buckling is characterized by K_y = 1.06. The compression force in the column prevents it from providing rotational support to the beam. Conversely, it is the column that will need the support of the beam which makes the situation worse for the beam than a simple pinned support would have been.

Flexural buckling parameters of both members are properly approximated by the AutoN_{cr} Tool.

4.3.3 Inclined supporting beam

The angle between the connected elements also influence their flexural buckling properties. This is illustrated in this example. The beam from the previous examples is rotated in both the buckling plane (β) and the horizontal plane (α) by 30 degrees (*Figure 25a*). This reduces its contribution as a rotational support for the column. Note that the beam section is modified to a circular hollow section (CHS) to avoid weak-axis buckling of IPE sections. This was considered more realistic than rotating an IPE section by 90 degrees to achieve the same effect. IPE weak-axis buckling is handled appropriately by the AutoNcr Tool, but it occurs practically independently of the strong-axis buckling of the column, thus, its investigation is of limited use for this example.

The results in *Figure 25b* and *Table 14* show that the compression force and the rotation of the beam significantly reduced its contribution as a rotational support. Using the same CHS beam without compression load on it would lead to a column K_y = 0.82 and N_{cr} = 7156 kN. If the beam is not inclined, but in the conventional right-angle position, the parameters are even more favorable: K_y = 0.80 and N_{cr} = 7430 kN. Further rotation of the beam farther from the buckling plane of the column, rapidly reduces its contribution and N_{cr} for the column.

The AutoN_{cr} Tool approximates the flexural buckling properties of both components appropriately. The results of this example confirm that the tool can take the inclination of connecting members into account and properly reduce the corresponding N_{cr} value.
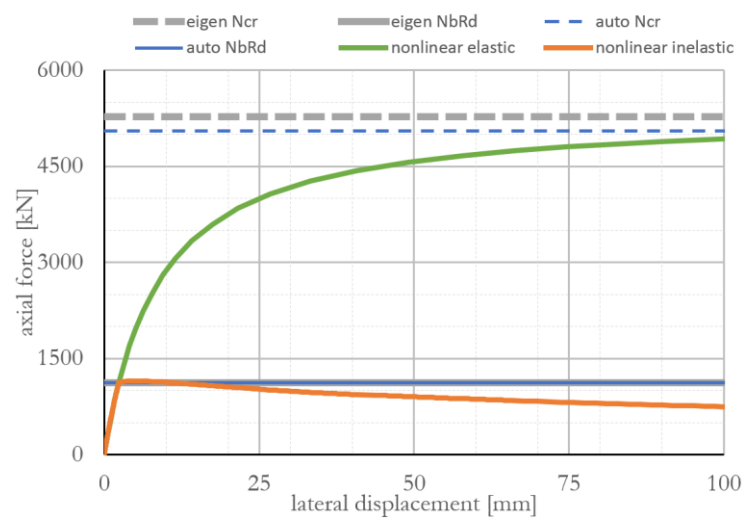
a)
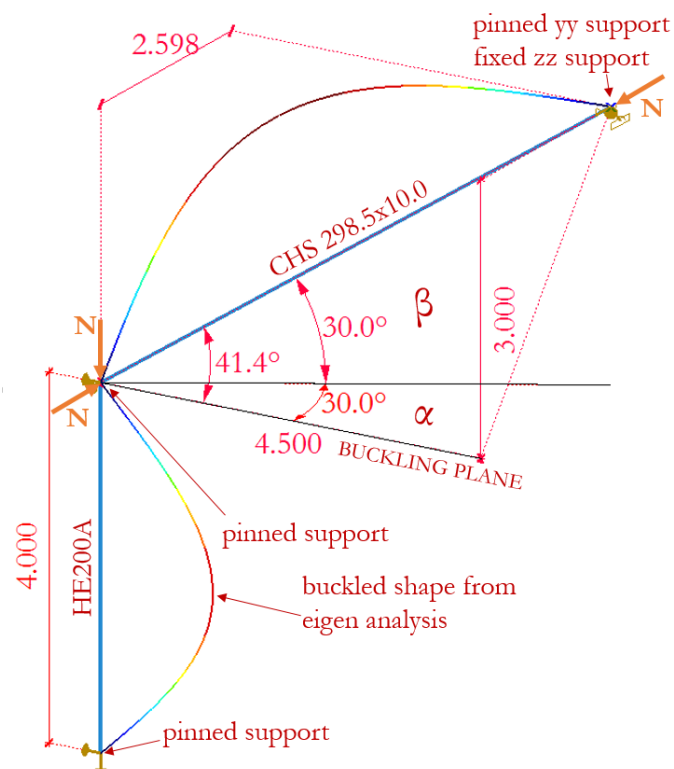
b)

Figure 25 *Flexural buckling analysis of a column supported by an inclined beam. a) Input data and buckled shape; b) axial force - maximum lateral displacement plots for different analyses.*

Table 14 Result summary for flexural buckling analysis of a column supported by an inclined beam. Forces are in kN. (N_bRd values in parenthesis correspond to global structural failure. The given element would not necessarily fail at those load levels)

		K_y	N_{cr}	N_{bRd}
BEAM	**eigen**	1.01	5277	1864
	AutoN_cr	1.01	5263	1863
	nonlinear elastic	-	5242	(1142)
	nonlinear inelastic	-	-	(1149)
COLUMN	**eigen**	0.95	5277	1125
	AutoN_cr	0.97	5053	1119
	nonlinear elastic	-	5242	1142
	nonlinear inelastic	-	-	1149

4.3.4 Accuracy: inclined supporting beams

This section demonstrates the accuracy of the AutoNcr Tool for columns supported by inclined beams. Various configurations are investigated based on the example in Section 4.3.2, but with no compression force in the beam. Because in-plane inclination does not have a strong influence on the flexural buckling parameters of non-sway columns, only three discrete β angles are considered. *Figure 26* shows that the out-of-plane slope (α) is more influential. As the beam is rotated in the horizontal plane, its rotational support gradually vanishes and the column becomes simply supported.

The gradual reduction of rotational support from the beam is appropriately followed by the AutoNcr Tool. The maximum error occurs at α = 75°, β = 0° where the difference compared to the reference eigen solution is less than 4%. This is considered acceptable for practical application. Note that neglecting the influence of beam inclination would lead to more than 10% error in K_y values.
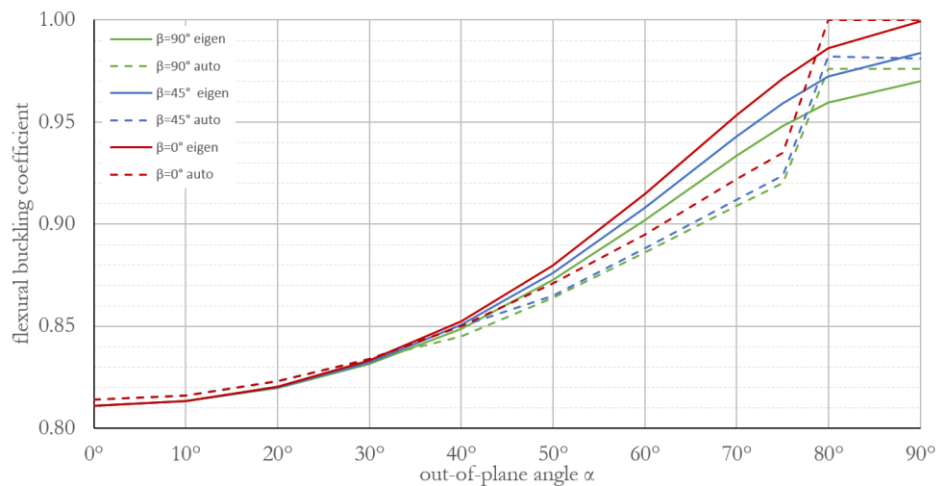

Figure 26 Flexural buckling coefficient of a column supported by an inclined beam at various in-plane (β) and out-of-plane (α) slopes. Continuous lines show reference results based on eigenvalue analysis; dashed lines correspond to results of the AutoN_cr Tool.

4.3.5 Applicability limits: tapered supporting beams

We have already demonstrated that the AutoN_cr Tool cannot always provide accurate solutions for members with non-uniform cross-sections (see Section 4.2.7 for details). Although discrete jumps in cross-sections are allowed, but not recommended, automatic flexural buckling parameter calculation for tapered members is

completely disabled. However, the support provided by a tapered beam shall be taken into consideration for the analysis of other members. The corresponding limitations are explained here through an example.

The example is similar to the previous ones, but the beam has a variable cross-section: it starts as an IPE500 and ends in an IPE300 (*Figure 27*). The current version of the AutoN_{cr} Tool models this beam with a uniform cross-section. In order to be on the conservative side, the properties of the approximate beam are defined by the tool to accurately model the less favorable scenario: when the smaller section is connected to the column under consideration. This is highlighted by the results in *Table 15*. The first case is the one shown in *Figure 27*, while the beam is in the opposite direction in the second case. Note that the automatic calculation provides the same result for both cases and its result is close to the eigen result of the less favorable case.
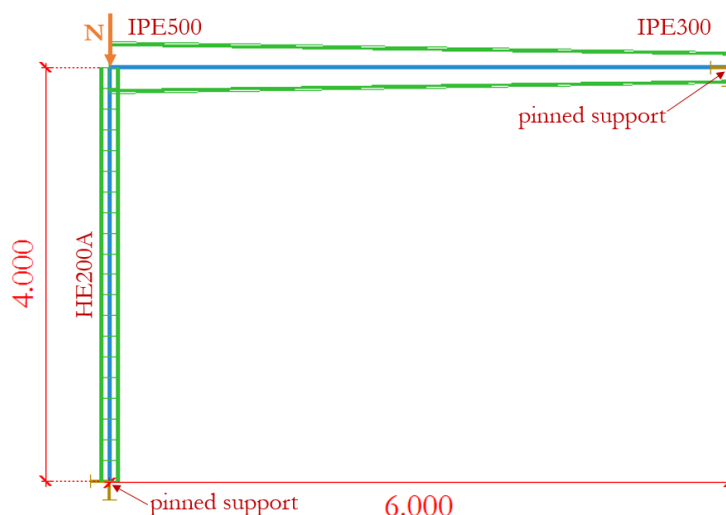

Figure 27 *Input data for flexural buckling analysis of a column supported by a tapered beam.*

Table 15 *Result summary for flexural buckling analysis of a column supported by a tapered beam. Forces are in kN.*

		K_y	N_{cr}	N_{bRd}
COLUMN	**eigen**	0.74	8787	1181
IPE500 > IPE300	**AutoN$_{cr}$**	0.79	7743	1170
COLUMN	**eigen**	0.78	7850	1171
IPE300 > IPE500	**AutoN$_{cr}$**	0.79	7743	1170

Consequently, columns connected to the smaller end of a tapered beam are expected to get accurate estimates for their flexural buckling parameters. Calculation of columns supported by the higher end of a tapered beam will be somewhat conservative, but the error (especially in the N_{bRd} value) is considered acceptable.

5. STRUCTURAL EXAMPLE

After detailed analysis of several single member configurations, the following example shows a structural model closer to everyday design practice. It highlights a few important settings in such cases that are needed to get appropriate results with the AutoN_{cr} Tool.

The example structure is the main frame of an industrial hall. *Figure 28* displays details of the numerical model. The frame is made up from two HE400A columns connected by an IPE500 beam. The beam is laterally supported at several points by a bracing system. The braces are modelled as truss members with pinned supports on their other ends. The lateral supports are assumed to be sufficiently close to each other to prevent lateral torsional buckling of the beam. Columns are fixed in-plane and pinned out-of-plane at their base. The beam-column connection is assumed to behave a perfectly rigid joint.

A total of three load configurations are investigated. Both columns are loaded by the same normal force, while the beam receives no compression in the first case (*Figure 28*). Conversely, the second configuration puts uniform compression force on the beam and no axial force on the columns (*Figure 31*). The third configuration is a combination of the previous cases: both columns and the beam are loaded by uniform compression to show how the interaction of the two elements is handled by the AutoN_{cr} Tool.

Results for the first scenario are shown in *Figures 29* and *30*. Out-of-plane buckling is simple in this case, because only the columns buckle and they behave as simply supported members. Because the bracing system provides ample support for the main frame against out-of-plane translation, the column is set non-sway in the local x-y plane. The AutoN_{cr} Tool provides exact results for this case.

In-plane buckling is more challenging, because of the finite support provided by the beam to the columns. The top of the columns is not supported against translation; second order effects are expected to have significant influence on the buckling resistance. Therefore, the columns are set sway in the local x-z plane. Results of the AutoNcr tool approximate the eigen results with minimal error.

The second loading scenario is displayed in *Figure 31*, while the corresponding results are shown in *Figures 32* and *33*. Out-of-plane buckling of the beam is also a basic problem, but its proper approximation requires the AutoN_{cr} tool to recognize lateral supports provided by the bracing members. The beam is set as non-sway for buckling in the local x-y plane. Results in *Figure 32* show that the automatic calculation provides exact parameters for this case: the buckling length equals the distance between two lateral supports.

In-plane buckling of the beam is displayed in *Figure 33*. Because the columns do not receive axial load, they will provide translational support to the beam ends. Therefore, the beam is considered non-sway for buckling in the local x-z plane. The support of bracing members is neglected for in-plane buckling analysis. The beam is not straight in the model, but the AutoN_{cr} Tool assumes straight members for its calculations. This leads to a small error on the conservative side for the beam. The 3% error in N_{bRd} is considered acceptable.

Results of the third scenario for out-of-plane buckling are identical to the previous ones for column and beam flexural buckling. Neither member is supported by the other against out-of-plane flexural buckling. Thus, the combined loading does not modify their out-of-plane buckling behavior.
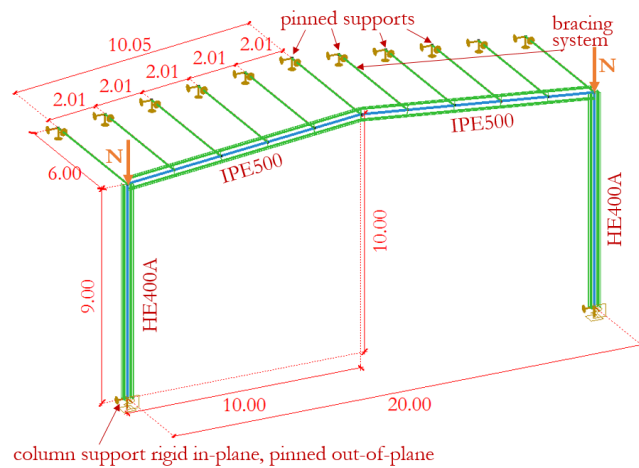
Figure 28 *Input data and numerical model for flexural buckling analysis of the main frame of an industrial hall. This first loading scenario model axial loads only for the columns.*
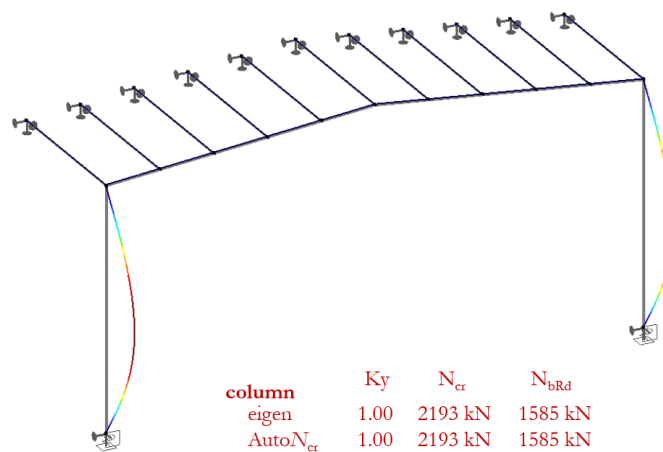

column	K_y	N_{cr}	N_{bRd}
eigen	1.00	2193 kN	1585 kN
AutoN_{cr}	1.00	2193 kN	1585 kN

Figure 29 *Buckled shape of the structure corresponding to out-of-plane flexural buckling from column-only loading and result summary for eigen and automatic analyses.*

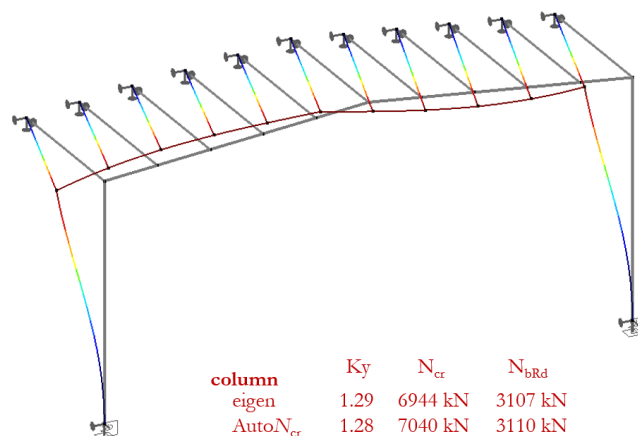| column	K_y	N_{cr}	N_{bRd}
eigen	1.29	6944 kN	3107 kN
AutoN_{cr}	1.28	7040 kN	3110 kN

Figure 30 *Buckled shape of the structure corresponding to in-plane flexural buckling from column-only loading and result summary for eigen and automatic analyses.*
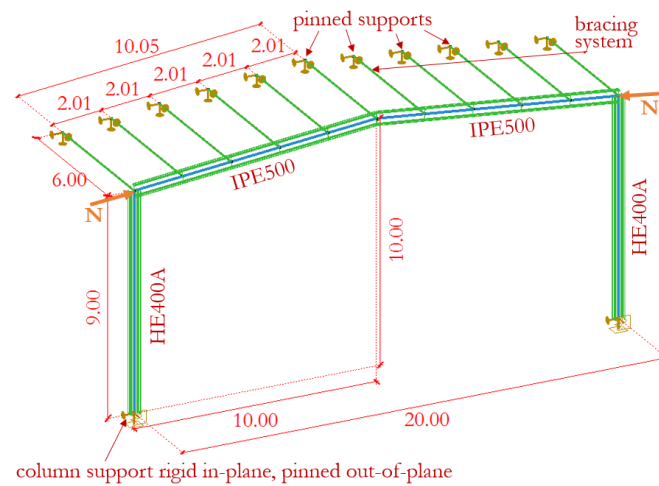
Figure 31 *Input data and numerical model for flexural buckling analysis of the main frame of an industrial hall. This second loading scenario model axial loads only for the beam.*
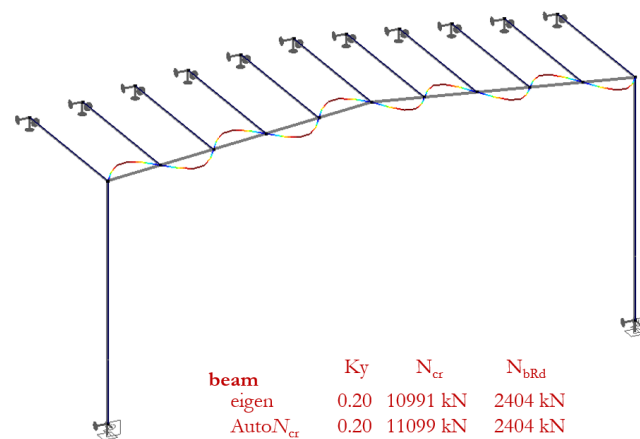

beam	K_y	N_{cr}	N_{bRd}
eigen	0.20	10991 kN	2404 kN
$AutoN_{cr}$	0.20	11099 kN	2404 kN

Figure 32 *Buckled shape of the structure corresponding to out-of-plane flexural buckling from beam-only loading and result summary for eigen and automatic analyses.*

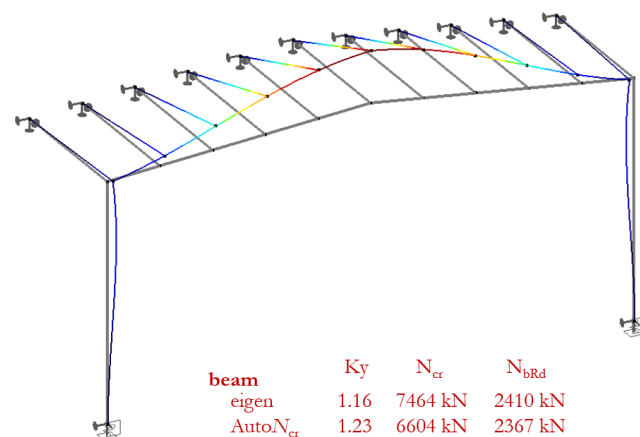| beam	K_y	N_{cr}	N_{bRd}
eigen	1.16	7464 kN	2410 kN
$AutoN_{cr}$	1.23	6604 kN	2367 kN

Figure 33 *Buckled shape of the structure corresponding to in-plane flexural buckling from beam-only loading and result summary for eigen and automatic analyses.*

In-plane buckling on the other hand is strongly affected by the combination of axial loads. Although the buckled shape in *Figure 34* is very similar to the one in *Figure 30* that corresponds to column buckling, the corresponding N_{cr} and K_y values are less favorable. Columns shall obviously be set sway in-plane, but the settings for the beam are not so trivial. The proper setting depends on the relative rigidity and strength of the beam and the columns.

If the columns can support the beam at the critical load level that leads to in-plane flexural buckling, then the beam shall be set non-sway. This happens if the beam is the critical component and the structure fails by experiencing excessive plastic deformations at one of the beam cross-sections. In such cases N_{Ed} of columns is often far from their flexural buckling resistance.

More often the beam is the stronger component and the structure fails by plastic deformation of one or both of its columns. In such structures in-plane flexural buckling will lead to significant vertical translation of the beam-column joints. This causes second-order moments in the beam thus its flexural buckling shall be classified as sway. This setting will increase K_y and decrease N_{bRd} significantly for the beam.

Note that beams are typically designed for distributed vertical load and corresponding shear forces and bending moments. Thus, relatively small axial forces develop in them. Column buckling in such cases almost always occur before beam buckling, because N_{Ed} in beams is still small when N_{Ed} in the columns have reached their flexural buckling resistance. According to the reasoning above, such beams should have small corresponding N_{cr} and large K_y values and shall be classified as sway. However, the resulting small N_{bRd} values shall not be alarming, because N_{Ed} is small in the beams and their flexural buckling in such cases is not going to govern structural failure. Should the beam become the critical component, its buckling will no longer be characterized as sway. Therefore, from a practical viewpoint, when a large number of load combinations are analyzed and the user prefers to use a single setting for the beam for all load combinations, it is recommended to set the beam as non-sway. This will lead to conservative results in the non-governing cases but provide proper results in the governing ones.

The frame structure under investigation is loaded by a significant normal force and the beam becomes the critical component of the structure. This is taken into consideration by setting the beam as non-sway. The resulting flexural buckling parameters from the AutoN_{cr} Tool in *Figure 34* are sufficiently accurate approximations of the eigen results for both members.
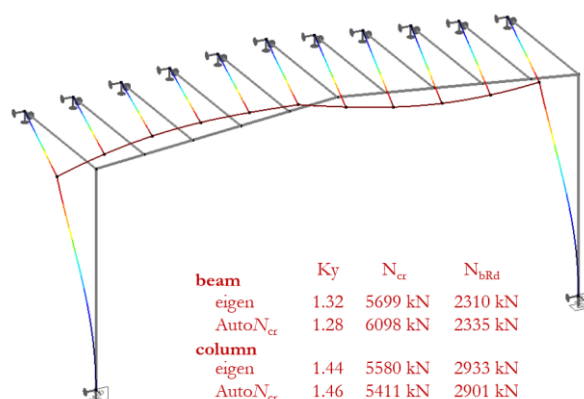

beam	Ky	N_{cr}	N_{bRd}
eigen	1.32	5699 kN	2310 kN
AutoN_{cr}	1.28	6098 kN	2335 kN
column			
eigen	1.44	5580 kN	2933 kN
AutoN_{cr}	1.46	5411 kN	2901 kN

Figure 34 *Buckled shape of the structure corresponding to in-plane flexural buckling from combined loading and result summary for eigen and automatic analyses.*

6. SUMMARY

This guide shall provide sufficient information for users of AxisVM to get familiar with the AutoN_{cr} Tool. The theoretical background presented in Sections 2 and 3 explain that the tool was designed to handle a specific set of problems and will not be able to provide an appropriate solution for every possible structural model. It is meant to provide sufficiently accurate estimates of flexural buckling parameters for a large set of structural configurations that are frequently used by practicing engineers. The developers hope that AxisVM users will find that the AutoN_{cr} Tool makes their work more efficient and their designs more economical. We are committed to the continuous improvement of this tool based on the feedback from its users.

The examples in Section 4 and 5 explain the working mechanisms and highlight typical pitfalls of stability analysis in general and the AutoN_{cr} Tool in particular. The single member case-studies cover the majority of practical problems including special sections that explain the applicability limits of the AutoN_{cr} Tool. This is part of our commitment to transparency: we prefer our users to know these limits and be able to use our tool wisely. The structural example in Section 5 illustrates that the tool can handle moderately complicated structures with various loading scenarios. The set of examples is expected to grow over time to cover all issues that users typically face based on their feedback.

7. REFERENCES

Chladný E. and Štujberová M., 2013, Frames with unique global and local imperfection in the shape of the elastic buckling mode (Part 1), Stahlbau, 82:8 609-17

ECCS Technical Committee 8 - Stability, 2006, Rules for Member Stability in EN 1993-1-1 Background documentation and design guidelines, European Convention for Constructional Steelwork (ECCS)European Committee for Standardization (CEN), 2009a, EN 1993-1-1:2009 Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings, CEN

European Committee for Standardization (CEN), 2009b, EN 1993-1-5:2009 Eurocode 3: Design of steel structures - Part 1-5: Plated structural elements, CEN